6.2&6.3: Separation of Variables/Differential Equations

The rate of change of y is directly proportional to t.

multiply

K: constant of proportionality

$$y' = kt$$

$$\int \frac{dy}{\cancel{dt}} = \int kt\,dt$$

$$y = \frac{kt^2}{2} + C \quad \text{or} \quad y = kt^2 + C$$

The rate of change of y is inversely proportional to the square root of t

divide

$$y' = \frac{k}{\sqrt{t}}$$

$$\int \frac{dy}{\cancel{dt}} = kt^{-1/2}\,dt$$

$$y = \frac{kt^{1/2}}{1/2} + C$$

$$y = 2kt^{1/2} + C \quad \text{or}$$

$$y = kt^{1/2} + C$$

The rate of change of y is proportional to y.

$$y' = ky$$

$$\frac{dy}{dt} = \frac{ky}{1}$$

$$\int \frac{dy}{y} = \int k\,dt$$

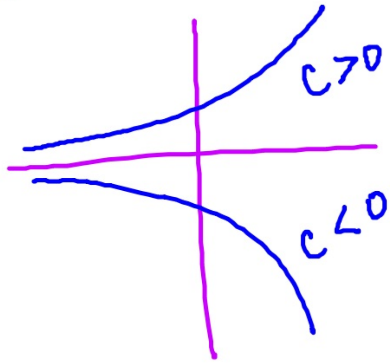
$$e^{\ln|y|} = e^{kt + c}$$

$$|y| = e^{kt} \cdot e^{c}$$

Directly proportional = proportional

$$e^{c} = C$$

$$y = Ce^{kt}$$



If $dy/dx = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

$(1, -1)$

(A) −2/3 (B) −1/3 (C) 0 (D) 1/3 (E) 2/3

$$\frac{dy}{dx} = 2y^2$$

$$\int \frac{dy}{y^2} = \int 2\,dx$$

$$\frac{y^{-1}}{-1} = 2x + C$$

$$-\frac{1}{y} = 2x + C$$

$(1, -1)$

$$1 = 2 + C$$

$$-1 = C$$

$$-\frac{1}{y} = \frac{2x - 1}{1}$$

$$-y = \frac{1}{2x-1}$$

$$y = \frac{-1}{2x-1}$$

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

t : hours
y : # bacteria

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

$y' = Ky$

$y = Ce^{Kt}$

$(0, 1)$

$(3, 2)$

$(0, 1)$

$1 = Ce^{0}$

$1 = C$

$y = e^{Kt}$

$(3, 2)$

$\ln 2 = \ln e^{3K}$

$\ln 2 = 3K$

$\frac{\ln 2}{3} = K$

$y = e^{\frac{\ln 2}{3}t}$

$\ln 3 = \ln e^{\frac{\ln 2}{3}t}$

$\ln 3 = t\frac{\ln 2}{3}$

$\frac{\ln 3}{\frac{1}{3}\ln 2} = t$

$\frac{3\ln 3}{\ln 2} = t$

If $dy/dx = x^2y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}}+7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3}+1$

$$\int \frac{dy}{y} = \int x^2\,dx$$

$$e^{\ln|y|} = e^{\frac{1}{3}x^3 + C}$$

$$y = e^{\frac{1}{3}x^3}(e^C)$$

$$y = Ce^{\frac{1}{3}x^3}$$

x^{2+3}

$x^2 \cdot x^3$

At each point on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point (0,8), then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

$$y' = 3x^2 y$$

$$\frac{dy}{dx} = 3x^2 y$$

$$\int \frac{dy}{y} = \int 3x^2\,dx$$

$$e^{\ln|y|} = e^{x^3 + C}$$

$$y = Ce^{x^3} \quad (0,8)$$

$$8 = Ce^0$$

$$8 = C$$