6.2&6.3: Separation of Variables/Differential Equations

(1) $\frac{dy}{dx} = \frac{3y}{x^2}$

$\int \frac{dy}{y} = \int \frac{3}{x^2} dx$

$\ln|y| = \int 3x^{-2} dx$

$\ln|y| = \frac{3x^{-1}}{-1} + C$

$e^{\ln|y|} = e^{\frac{-3}{x} + C}$

Find the general solution to the differential equation.

$|y| = e^{-3/x} \cdot e^{C}$

$y = Ce^{-3/x}$

Find the general solution to the differential equation.

② $y' - e^y \cos x = 0$

$\frac{dy}{dx} = e^y \cos x$

$\int \frac{dy}{e^y} = \int \cos x \, dx$

$u = -y$

$du = -1 dx$

$\int e^{-y} dy = \sin x + C$

$-\int e^u du$

$-1(-e^{-y} = \sin x + C)$

$\ln(e^{-y}) = \ln(-\sin x + C)$

$-y = \ln|-\sin x + c|$

$y = -\ln|-\sin x + c|$

Solve the differential equation.

(3) $\sqrt{1-x^2}\,y' - x = 0$

$$\sqrt{1-x^2}\frac{dy}{dx} = x$$

$$\int dy = \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$$y = -\frac{1}{2}\int u^{-1/2}\,du$$

$$y = -\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C$$

$$y = -\sqrt{1-x^2} + C$$

(4) $y^2y' - x^2 = 0$

Solve the differential equation given the initial condition.

$(1, 2)$

$y^2 \frac{dy}{dx} = x^2$

$\int y^2 dy = \int x^2 dx$

$\frac{y^3}{3} = \frac{x^3}{3} + C$

$\frac{8}{3} = \frac{1}{3} + C$

$\frac{7}{3} = C$

$3\left(\frac{y^3}{3} = \frac{x^3}{3} + \frac{7}{3}\right)$

$y^3 = x^3 + 7$

$y = \sqrt[3]{x^3 + 7}$

(5) $y' - (x+3)y^2 = 0$

$$\frac{dy}{dx} = (x+3)y^2$$

$$\int \frac{dy}{y^2} = \int (x+3)\,dx$$

$$\int y^{-2}\,dy = \frac{x^2}{2} + 3x + C$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 3x + C$$

$$-\frac{1}{y} = \frac{x^2}{2} + 3x + C$$

Solve the differential equation given the initial condition (0, 1)

$(0, 1)$

$$-1 = C$$

$$-\frac{1}{y} = \frac{x^2}{2} + 3x - 1$$

$$-\frac{1}{y} = \frac{x^2 + 6x - 2}{2}$$

$$-y = \frac{2}{x^2 + 6x - 2}$$

$$y = \frac{-2}{x^2 + 6x - 2}$$