# 6.2&6.3: Separation of Variables/Differential Equations

Quiz tomorrow: Rate in/Rate Out (with graphing calculator)

Quiz Monday: Slope fields and Differential Equations

The rate of change of y is directly proportional to x.

$K$: constant of proportionality

$$\frac{dy}{dx} = Kx \qquad y = \frac{Kx^2}{2} + C$$

$$\int dy = \int Kx\,dx$$

or

$$y = Kx^2 + C$$

The rate of change of y is inversely proportional to the square root of x

$$\frac{dy}{dx} = \frac{K}{\sqrt{x}} \qquad y = \frac{K \cdot x^{1/2}}{1/2} + C$$

$$\int dy = \int Kx^{-1/2}\,dx$$

$$y = 2K\sqrt{x} + C \quad \text{or} \quad y = K\sqrt{x} + C$$

The rate of change of y is proportional to y.

$$\frac{dy}{dx} = ky$$

$$\int \frac{dy}{y} = \int k\,dx$$

$$e^{\ln|y|} = e^{kx+C}$$

$$|y| = Ce^{kx}$$

$$y = Ce^{kx}$$

$$e^{kx+c} = e^{kx} \cdot e^{c}$$

$$= e^{kx} \cdot C$$

$$= Ce^{kx}$$

1 $\dfrac{dy}{dx} = \dfrac{3y}{x^2}$

Find the general solution to the differential equation.

$$\int \frac{dy}{y} = \int \frac{3}{x^2}\,dx = \int 3x^{-2}\,dx$$

$$\ln|y| = \frac{3x^{-1}}{-1} + C$$

$$\boxed{y = Ce^{-3/x}}$$

$$e^{\ln|y|} = e^{\frac{-3}{x} + C}$$

$$e^{(-3/x + C)} = e^{-3/x} \cdot \underbrace{e^{C}}_{C}$$

Find the general solution to the differential equation.

② $y' - e^{y}\cos x = 0$

$$\frac{dy}{dx} = e^{y}\cos x$$

$$\int \frac{dy}{e^{y}} = \int \cos x \, dx$$

$$\int e^{-y}\,dy = \sin x + C$$

$$-e^{-y} = \sin x + C$$

$$\ln\left(e^{-y}\right) = \ln\left(-\sin x + C\right)$$

$$-y = \ln\left|-\sin x + c\right|$$

$$y = -\ln\left|-\sin x + c\right|$$

③ $\sqrt{1-x^2}\,y' - x = 0$

Solve the differential equation.

$$\int dy = \int \frac{x}{\sqrt{1-x^2}}\,dx \qquad u = 1-x^2$$
$$du = -2x\,dx$$

$$y = -\frac{1}{2}\int u^{-1/2}\,du$$

$$y = -\frac{1}{2}\cdot\frac{u^{1/2}}{1/2} + C = -\sqrt{1-x^2} + C$$

④ $y^2 y' - x^2 = 0$

Solve the differential equation given the initial condition. $(0,2)$

$$y^2 \frac{dy}{dx} = x^2$$

$$\int y^2 dy = \int x^2 dx$$

$$\frac{y^3}{3} = \frac{x^3}{3} + C$$

$(0,2)$

$$\frac{8}{3} = 0 + C$$

$$\frac{8}{3} = C$$

$$3\left(\frac{y^3}{3} = \frac{x^3}{3} + \frac{8}{3}\right)$$

$$y^3 = x^3 + 8$$

$$y = \sqrt[3]{x^3 + 8}$$

$$3\left(\frac{y^3}{3} = \frac{x^3}{3} + C\right)$$

$$y^3 = x^3 + C$$

$$8 = 0 + C$$

$$8 = C$$

$$y^3 = x^3 + 8$$

$$y = \sqrt[3]{x^3 + 8}$$

5 $y' - (x+3)y^2 = 0$

Solve the differential equation given the initial condition.

$(0,1)$

$$\int \frac{dy}{y^2} = \int (x+3)\,dx$$

$$\int y^{-2}\,dy = \frac{1}{2}x^2 + 3x + C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + 3x + C$$

$$-1 = C$$

$$-1 = C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + 3x - 1$$

$$\frac{1}{y} = -\frac{x^2}{2} - 3x + 1$$

$$\frac{1}{y} = \frac{-x^2 - 6x + 2}{2}$$

$$y = \frac{2}{-x^2 - 6x + 2}$$

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\frac{3\ln 3}{\ln 2}$ (B) $\frac{2\ln 3}{\ln 2}$ (C) $\frac{\ln 3}{\ln 2}$ (D) $\ln\left(\frac{27}{2}\right)$ (E) $\ln\left(\frac{9}{2}\right)$

$y = Ce^{kt}$ $(0, 1)$

$(3, 2) \rightarrow 2 = 1 \cdot e^{k \cdot 3}$

$1 = Ce^{0}$

$1 = C$

$y = 1 \cdot e^{\frac{1}{3}\ln 2 \cdot t}$

$\ln 3 = \ln e^{\frac{1}{3}\ln 2 t}$

$\ln 3 = \frac{1}{3}\ln 2 \cdot t$

$\ln 2 = \ln e^{3k}$

$\frac{\ln 2}{3} = k$

If $dy/dx = x^2y$, then $y$ could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}}+7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3}+1$

$$\int \frac{dy}{y} = \int x^2\,dx$$

$$e^{\ln|y|} = e^{\frac{1}{3}x^3 + C}$$

$$y = Ce^{x^3/3}$$

If $dy/dx = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-2/3$ (B) $-1/3$ (C) $0$ (D) $1/3$ (E) $2/3$

$$\int \frac{dy}{y^2} = \int 2\,dx$$

$$-\frac{1}{y} = 2x + C$$

$$1 = 2 + c$$

$$-1 = c$$

$$-\frac{1}{y} = \frac{2x-1}{1}$$

$$-y = \frac{1}{2x-1}$$

$$y = \frac{-1}{2x-1}$$