5.7 Inverse Trigonometric Functions: Integration

- Integrate functions whose antiderivatives involve inverse trigonometric functions.
- Use the method of completing the square to integrate a function.
- Review the basic integration rules involving elementary functions.

The final list of techniques of integration:

power rule
u-substitution
ln x (u'/u)
long division (degree num > or = degree den)
inverse trig
completing the square

#1

$$\int \frac{4x+3}{\sqrt{1-x^2}}\,dx = \underbrace{\int \frac{4x}{\sqrt{1-x^2}}\,dx}_{u\text{-sub}} + \underbrace{\int \frac{3}{\sqrt{1-x^2}}\,dx}_{\text{arcsin}}$$

$u = 1 - x^2$

$du = -2x\,dx$

$$4 \cdot -\frac{1}{2} \int u^{-1/2}\,du$$

$$-2 \cdot \frac{u^{1/2}}{1/2}$$

$$-4\sqrt{1-x^2} + 3\arcsin x + C$$

#2 $\int \frac{dx}{x^2 - 4x + 7}$ $= \int \frac{dx}{(x-2)^2 + 3}$

Completing the square

arctan

$(x^2 - 4x + 4) + 7 - 4$

$u = x - 2$

$du = 1\,dx$

$a = \sqrt{3}$

$(x-2)^2 + 3$

$\frac{1}{\sqrt{3}} \arctan \frac{(x-2)}{\sqrt{3}} + C$

$$\int \frac{dx}{x^2-6x+19} = \int \frac{dx}{(x-3)^2+10}$$

$$\frac{1}{\sqrt{10}} \arctan \frac{x-3}{\sqrt{10}} + C$$

$$\int \frac{x+4}{x^2+25}\,dx = \int \frac{x}{x^2+25}\,dx + \int \frac{4}{x^2+25}\,dx$$

ln (u-sub) arctan

$$\frac{1}{2}\ln|x^2+25| + \frac{4}{5}\arctan\frac{x}{5} + C$$

#3 $\displaystyle\int \frac{2}{\sqrt{-x^2+4x}}\,dx$ $= \displaystyle\int \frac{2}{\sqrt{4-(x-2)^2}}\,dx$

$-(x^2-4x+4)+4$

$-(x-2)^2+4$

$u = x-2$

$du = 1\,dx$

$a = 2$

$2\arcsin\frac{x-2}{2} + C$

53.) $y' = 2x \cdot \frac{-1}{\sqrt{1-x^2}} + \arccos x \cdot 2 - \cancel{2} \cdot \frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-2x)$

$= \cancel{\frac{-2x}{\sqrt{1-x^2}}} + 2\arccos x + \cancel{\frac{2x}{\sqrt{1-x^2}}}$

43.) $y = 2\arcsin(x-1)$

$$y' = \frac{2}{\sqrt{1-(x-1)^2}}$$

$$y' = \frac{2}{\sqrt{1-(x^2-2x+1)}} = \frac{2}{\sqrt{-x^2+2x}}$$