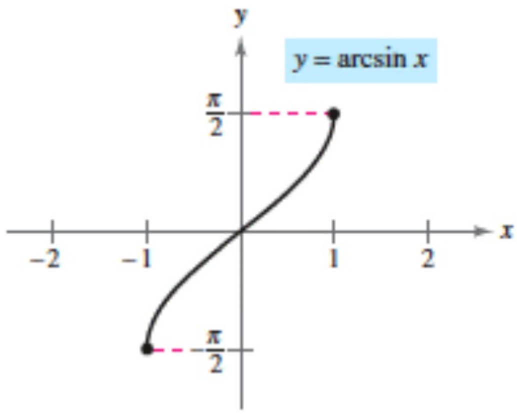
5.6 Inverse Trigonometric Functions: Differentiation

- Develop properties of the six inverse trigonometric functions.
- Differentiate an inverse trigonometric function.
- Review the basic differentiation rules for elementary functions.

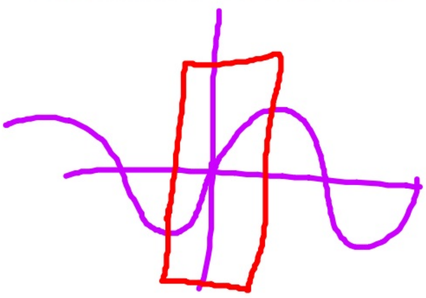
DEFINITIONS OF INVERSE TRIGONOMETRIC FUNCTIONS

Function	*Domain*	*Range*
$y = \arcsin x$ iff $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ iff $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ iff $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \text{arccot}\, x$ iff $\cot y = x$	$-\infty < x < \infty$	$0 < y < \pi$
$y = \text{arcsec}\, x$ iff $\sec y = x$	$\lvert x\rvert \ge 1$	$0 \le y \le \pi, \quad y \ne \frac{\pi}{2}$
$y = \text{arccsc}\, x$ iff $\csc y = x$	$\lvert x\rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, \quad y \ne 0$



Domain: $[-1, 1]$

Range: $[-\pi/2, \pi/2]$



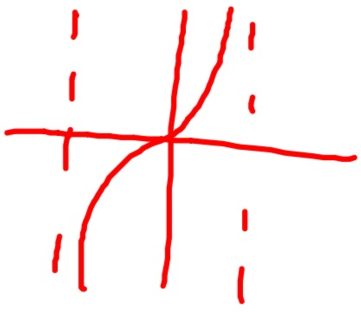
$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$[0, \pi]$ π

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\arctan(-1) = -\frac{\pi}{4}$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

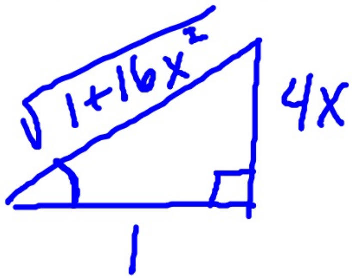


$$\tan\left(\arctan(2x-5)\right) = \tan\left(\frac{\pi}{4}\right)$$

$$2x-5=1$$

$$x=3$$

$$\sec(\arctan 4x)$$

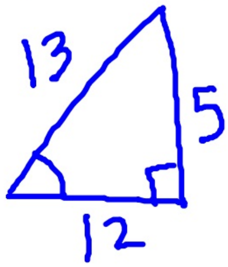


$$1^2+(4x)^2=c^2$$

$$\sqrt{1+16x^2}=c$$

$$\boxed{\sqrt{1+16x^2}}$$

$$\cos\left(\arcsin\frac{5}{13}\right)$$



$$\frac{12}{13}$$

Find y' for y = arcsin x

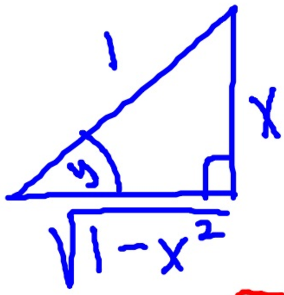
$$\sin y = \sin(\arcsin x)$$

$$\frac{d}{dx}\left(\sin y = x\right)$$

$$\cos y \cdot \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

$$y' = \frac{1}{\sqrt{1-x^2}}$$



$$\cos y = \frac{\sqrt{1-x^2}}{1}$$

$$f(t) = \arcsin t^2$$

$$f'(t) = \frac{2t}{\sqrt{1-t^4}}$$

$$y = \arcsin u$$

$$y' = \frac{u'}{\sqrt{1-u^2}}$$

$$y' = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

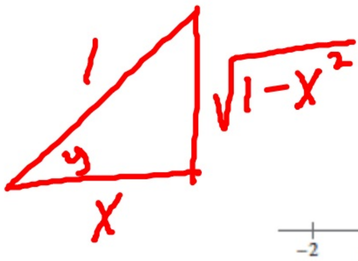
Find y' for y = arccos x

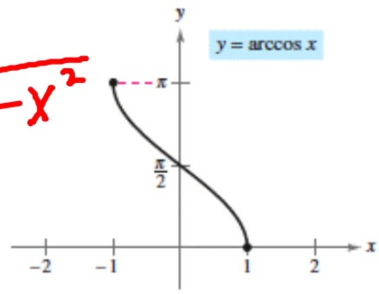
$$\left(\cos y = x\right)\frac{d}{dx}$$

$$-\sin y\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{-1}{\sin y}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

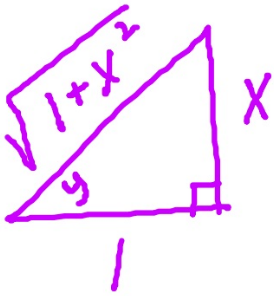


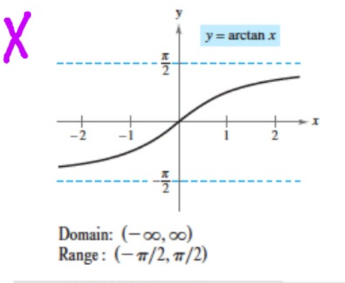


Domain: [−1, 1]
Range: [0, π]

Find y' for y = arctanx

$$\left(\tan y = x\right)\frac{d}{dx}$$



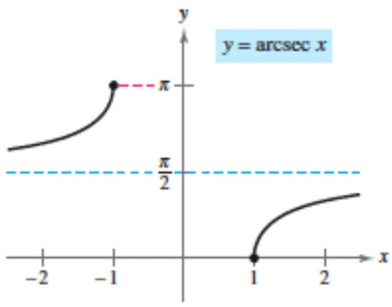


$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y = \left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Find y' for y = arcsecx



Domain: $(-\infty, -1] \cup [1, \infty)$
Range : $[0, \pi/2) \cup (\pi/2, \pi]$

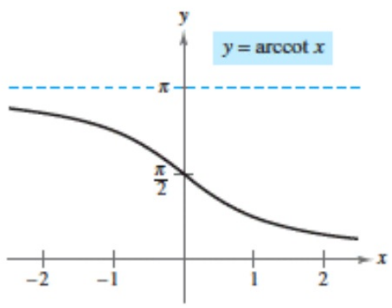
THEOREM 5.16 DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

Let u be a differentiable function of x.

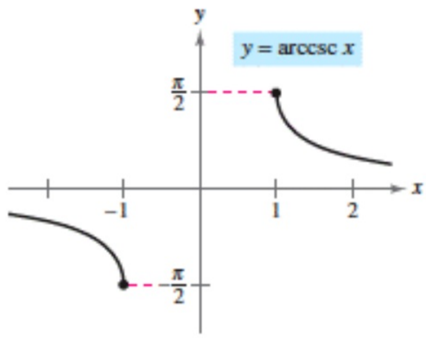
$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$



Domain: $(-\infty, \infty)$
Range: $(0, \pi)$



Domain: $(-\infty, -1] \cup [1, \infty)$
Range: $[-\pi/2, 0) \cup (0, \pi/2]$
Figure 5.29

Find the equation of the tangent line.

$$y = \arctan \frac{x}{2} \quad \left(2, \frac{\pi}{4}\right)$$

$$y' = \frac{\frac{1}{2}}{1 + \frac{x^2}{4}}$$

$$y' = \frac{2}{4+x^2}$$

$$y' = \frac{1}{4} \qquad \boxed{y - \frac{\pi}{4} = \frac{1}{4}(x-2)}$$

Find f'(x)

$$f(x) = \text{arcsec } 2x$$

$$f'(x) = \frac{2}{|2x|\sqrt{4x^2-1}}$$

$$f'(x) = \frac{1}{|x|\sqrt{4x^2-1}}$$

Find f'(x)

$$f(x) = \arcsin\left(\frac{x}{4}\right)$$

$$f'(x) = \frac{\frac{1}{4}}{\sqrt{1-\frac{x^2}{16}}} = \frac{\frac{1}{4}}{\sqrt{\frac{16-x^2}{16}}} = \frac{\frac{1}{4}}{\frac{\sqrt{16-x^2}}{4}}$$

$$= \frac{1}{\sqrt{16-x^2}}$$

Find f'(x)

Chain Rule

$$f(x) = \cos(\arcsin x)$$

$$f'(x) = -\sin(\arcsin x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{-x}{\sqrt{1-x^2}}$$

Find f'(x)

triangles

$$f(x) = \cos(\arcsin x)$$

$$f(x) = \sqrt{1-x^2}$$

$$f'(x) = \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$$

$$= \frac{-x}{\sqrt{1-x^2}}$$