5.5 Bases Other Than *e* and Applications

- Define exponential functions that have bases other than *e*.
- Differentiate and integrate exponential functions that have bases other than *e*.
- Use exponential functions to model compound interest and exponential growth.

#1 $y = \log_3(x^2 - 3x)$

Use the change of base formula and use the rules of the natural log to find y'.

$$y = \frac{\log(x^2-3x)}{\log 3}$$

$$y = \frac{\ln(x^2-3x)}{\ln 3} = \frac{1}{\ln 3} \cdot \ln(x^2-3x)$$

$$y' = \frac{1}{\ln 3} \cdot \frac{2x-3}{x^2-3x}$$

$$y = \log_a u$$

$$y' = \frac{1}{\ln a} \cdot \frac{u'}{u}$$

Find the equation of a tangent line through the given point.

#3 $y = \log_{10} 2x, \quad (5, 1)$

$y' = \frac{1}{\ln 10} \cdot \frac{2}{2x}$

$y' = \frac{1}{\ln 10} \cdot \frac{1}{x}$

$y'(5) = \frac{1}{5 \ln 10}$

$y - 1 = \frac{1}{5 \ln 10}(x - 5)$

$5 \ln 10 = \ln 10^5$

$= \ln 100000$

#2 $$g(x) = \log_5 \frac{4}{x^2\sqrt{1-x}}$$

$$g(x) = \log_5 4 - 2\log_5 x - \frac{1}{2}\log_5 (1-x)$$

$$g'(x) = \frac{1}{\ln 5}\left(0 - \frac{2}{x} - \frac{1}{2}\cdot\frac{-1}{1-x}\right)$$

$$\boxed{g'(x) = \frac{1}{\ln 5}\left(\frac{-2}{x} + \frac{1}{2(1-x)}\right)}$$

Find the derivative using logarithmic differentiation.

#4

$$y = x^{x-1}$$

Derivative of an exponential function.

#5 $f(x) = 3^x$

$\ln y = \ln 3^x$

$\frac{d}{dx}(\ln y = x \cdot \ln 3)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \ln 3 \cdot \cancel{\frac{dx}{dx}}$

$\frac{dy}{dx} = \ln 3 \cdot y$

$\frac{dy}{dx} = \ln 3 \cdot 3^x$

$y = a^x$

$y' = \ln a \cdot a^x$

$y = a^u$

$y' = \ln a \cdot a^u \cdot u'$

#6

$$f(t) = \frac{3^{2t}}{t}$$

$$f'(t) = \frac{t \cdot \ln 3 \cdot 3^{2t} \cdot 2 - 3^{2t} \cdot 1}{t^2}$$

$$= \frac{3^{2t}(2t \ln 3 - 1)}{t^2}$$

$$\ln y = \ln \frac{2x}{x-1}$$

$$\left(\ln y = \ln 2x - \ln(x-1)\right)\frac{d}{dx}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x-1}$$

$$\frac{dy}{dx} = \left(\frac{1}{x} - \frac{1}{x-1}\right)\cdot y = \left(\frac{1}{x} - \frac{1}{x-1}\right)\frac{2x}{x-1}$$

#7 $\displaystyle\int_{-2}^{2} 4^{x/2}\,dx$

$$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C$$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$$2\int_{-1}^{1} 4^u\,du = 2\cdot\frac{1}{\ln 4}\cdot 4^u \Big|_{-1}^{1}$$

$$= \frac{2}{\ln 4}\left(4^1 - 4^{-1}\right)$$

$$= \frac{2}{\ln 4}\left(4 - \frac{1}{4}\right) = \frac{2}{\ln 4}\cdot\frac{15}{4} = \frac{15}{2\ln 4} = \frac{15}{\ln 16}$$

115.) $\int e^{-x} \tan(e^{-x})\,dx$

$u = e^{-x}$

$du = -e^{-x}dx$

$-\int \tan u\, du$

$= -(-\ln|\cos u|) + C$

$= \ln|\cos(e^{-x})| + C$

79.) $f(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

$$f'(x) = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$= \frac{1}{2}\left(e^{x} - \frac{1}{e^{x}}\right)$$

$$= \frac{1}{2}\left(\frac{e^{2x} - 1}{e^{x}}\right)$$

$$e^{2x} - 1 = 0$$

$$\ln e^{2x} = \ln 1$$

$$2x = 0$$

$$x = 0$$