

5.4

Exponential Functions: Differentiation and Integration

- Develop properties of the natural exponential function.
- Differentiate natural exponential functions.
- Integrate natural exponential functions.

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x .

1. $\frac{d}{dx}[e^x] = e^x$

2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx} = e^u \cdot u'$

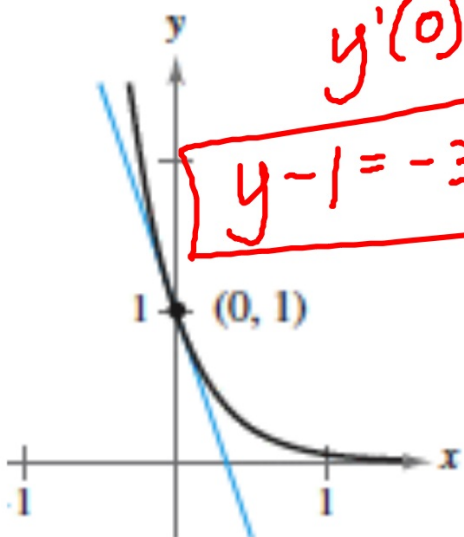
$$y = e^{x^2}$$
$$y' = e^{x^2} \cdot 2x$$

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

$$\ln e = 1 \quad \ln 1 = 0$$

#1: Find the equation of the tangent line at the given point $-3x$

$$y = e^{-3x}$$



$$y' = -3e^{-3x}$$
$$y'(0) = -3$$

$$y - 1 = -3(x - 0)$$

#2: Find y'

$$y = x^2 e^{-x}$$

$$y' = x^2 \cdot e^{-x}(-1) + e^{-x} \cdot 2x$$

$$y' = x e^{-x} (-x + 2)$$

or

$$y' = -x e^{-x} (x - 2)$$

#3: Find y' .

$$y = \frac{e^{2x}}{e^{2x} + 1}$$

$$y' = \frac{(e^{2x} + 1)2e^{2x} - e^{2x} \cdot 2e^{2x}}{(e^{2x} + 1)^2}$$

$$y' = \frac{\cancel{2e^{4x}} + 2e^{2x} - \cancel{2e^{4x}}}{(e^{2x} + 1)^2}$$

$$y' = \frac{2e^{2x}}{(e^{2x} + 1)^2}$$

#4: Find dy/dx

$$(e^{xy} + x^2 - y^2 = 10) \frac{d}{dx}$$

$$e^{xy} \left(x \frac{dy}{dx} + y \cdot 1 \right) + 2x - 2y \frac{dy}{dx} = 0$$

$$xe^{xy} \frac{dy}{dx} + ye^{xy} + 2x - 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}$$

$$3a.) \quad y = \ln \frac{e^{2x}}{e^{2x} + 1}$$

$$y = \ln e^{2x} - \ln(e^{2x} + 1)$$

$$y = 2x - \ln(e^{2x} + 1)$$

$$y' = \frac{2}{1} - \frac{2e^{2x}}{e^{2x} + 1} = \frac{2}{e^{2x} + 1}$$

$$y = \ln u,$$
$$y' = \frac{u'}{u}$$

$$y' = \frac{2e^{2x}}{(e^{2x} + 1)^2}$$
$$\frac{e^{2x}}{e^{2x} + 1}$$

THEOREM 5.12 INTEGRATION RULES FOR EXPONENTIAL FUNCTIONS

Let u be a differentiable function of x .

1. $\int e^x dx = e^x + C$ 2. $\int e^u du = e^u + C$

$$\begin{aligned} \int e^{4x} dx &= \frac{1}{4} \int e^u du \\ u=4x \\ du=4dx &= \frac{1}{4} e^u + C \\ &= \frac{1}{4} e^{4x} + C \end{aligned}$$

$$\#5 \int e^{1-3x} dx$$
$$u = 1-3x$$
$$du = -3 dx$$

$$-\frac{1}{3} \int e^u du$$

$$\frac{-1}{3} e^{1-3x} + C$$

$$\#6 \int \frac{e^{2x}}{1+e^{2x}} dx$$
$$u = 1+e^{2x}$$
$$du = 2e^{2x} dx$$

$$\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \int \frac{du}{u}$$

$$\frac{1}{2} \ln|u| + C$$

$$\frac{1}{2} \ln|1+e^{2x}| + C$$

#7

$$\int_0^1 \frac{e^x}{5 - e^x} dx$$

$$u = 5 - e^x$$
$$du = -e^x dx$$

$$-\int_4^{5-e} \frac{1}{u} du = -\ln|u| \Big|_4^{5-e}$$

$$= -\left(\ln|5-e| - \ln|4|\right) \checkmark$$

$$-\ln \frac{5-e}{4} = \ln \left(\frac{5-e}{4}\right)^{-1} \checkmark$$
$$= \ln \frac{4}{5-e} \checkmark$$