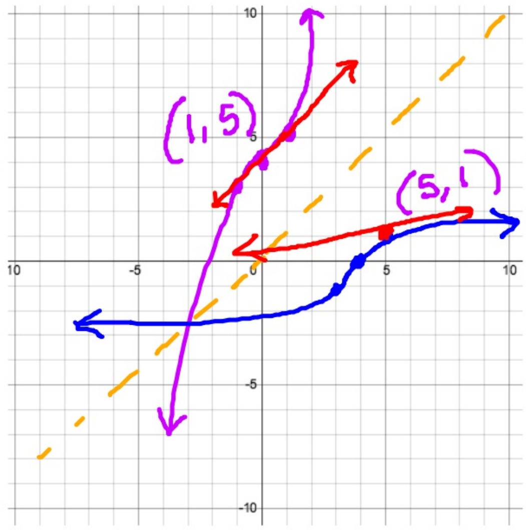
5.3 Inverse Functions

$f(x) = x^3 + 4 \qquad f^{-1}(x) = \sqrt[3]{x-4}$

$f'(x) = 3x^2 \qquad (f^{-1})'(x) = \frac{1}{3}(x-4)^{-2/3} \cdot 1$

$f'(1) = 3 \qquad (f^{-1})'(5) = \frac{1}{3}$



(1) $f(x) = 2x^3 + 3x$ $\qquad (f^{-1})'(5) =$ ______

Verify the function is 1:1 (monotonic: a function that is always increasing or always decreasing)

$f'(x) = 6x^2 + 3$

$+$ $\quad f'$

f(x) is monotomic because

f'>0

Find the derivative of the inverse at x = 5.

$5 = 2x^3 + 3x$

$x = 1$

$f'(x) = 6x^2 + 3$

$f'(1) = 9$

$(f^{-1})'(5) = \frac{1}{9}$

$f: (1, 5)$

$f^{-1}: (5, 1)$

THEOREM 5.9 THE DERIVATIVE OF AN INVERSE FUNCTION

Let f be a function that is differentiable on an interval I. If f has an inverse function g, then g is differentiable at any x for which $f'(g(x)) \neq 0$. Moreover,

$$g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0.$$

(2) $f(x) = x^3 - \frac{4}{x}$ $D: (0, \infty)$

f(x) and g(x) are inverses. Find g'(6)

$6 = x^3 - \frac{4}{x}$

$2 = x$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 13$

$g'(6) = \frac{1}{13}$

$f: (2, 6)$

$g: (6, 2)$

3) Let f be a differentiable function with $f(3) = 15$, $f'(3) = -8$, $f'(6) = -2$, $f(6) = 3$. The function g is differentiable and $g(x) = f^{-1}(x)$. What is the value of $g'(3)$?

$f'(6) = -2$

$f : (6, 3)$

$g : (3, 6)$

a) -1/8

b) -1/2

c) 1/6

d) 1/3

e) not possible

AP Question. Mean Score 0.95

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.