5.2: More Integration

#1: Evaluate the definite integral

$$\int_1^5 \frac{x+6}{x}\,dx = \int_1^5 \left(1 + \frac{6}{x}\right)dx$$

$$x + 6\ln|x| \Big]_1^5$$

$$(5 + 6\ln 5) - (1 + 6\ln 1)$$

$$4 + 6\ln 5 \text{ or } 4 + \ln 5^6$$

#2 $\frac{dy}{dx} = \frac{\sec^2 x}{1+\tan x}$

Solve the differential equation given the point (pi, 4)

$$\int dy = \int \frac{\sec^2 x}{1+\tan x}\,dx \qquad u = 1+\tan x \quad du = \sec^2 x\,dx$$

$$y = \int \frac{1}{u}\,du = \ln|u| + C$$

$$y = \ln|1+\tan x| + C \quad (\pi, 4)$$

$$4 = 0 + C$$

$$4 = C \qquad \boxed{y = \ln|1+\tan x| + 4}$$

#3

$$\int_{\pi/2}^{2\pi/3} \cot x \, dx = \int_{\pi/2}^{2\pi/3} \frac{\cos x}{(\sin x)'} dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int \frac{1}{u} du = \ln|u|$$

$$= \ln|\sin x| \Big]_{\pi/2}^{2\pi/3}$$

$$= \ln\frac{\sqrt{3}}{2} - \ln 1$$

$$= \ln\frac{\sqrt{3}}{2} = \ln\sqrt{3} - \ln 2$$

#4: Solve the differential equation given the initial condition.

$$\frac{ds}{d\theta} = \tan 2\theta, \quad (0, 2)$$

$$\int ds = \int \tan 2\theta \, d\theta$$

$u = \cos 2\theta$

$$s = \int \frac{\sin 2\theta}{\cos 2\theta} d\theta \qquad du = -2\sin 2\theta \, d\theta$$

$$s = -\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|\cos 2\theta| \cdot$$

$C = 2$

$$\boxed{s = -\frac{1}{2}\ln|\cos 2\theta| + 2}$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \quad \text{(proof pg. 338)}$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

#5 $y = x\sqrt{1-x^2}$ Find y' using log differentiation

$$\ln y = \ln\left(x\sqrt{1-x^2}\right)$$

$$\left(\ln y = \ln x + \frac{1}{2}\ln(1-x^2)\right)\frac{d}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\cdot\frac{-2x}{1-x^2}$$

$$\frac{dy}{dx} = \left(\frac{1}{x} + \frac{-x}{1-x^2}\right)x\sqrt{1-x^2}$$

$$= \left(\frac{1-x^2-x^2}{\cancel{x}(1-x^2)^1}\right)\cancel{x}(1-x^2)^{1/2} = \frac{1-2x^2}{\sqrt{1-x^2}}$$

67.) $F(x) = \int_1^x \frac{1}{t}\,dt$

$F'(x) = \frac{1}{x} \cdot 1$

43.) $\frac{dy}{dx} = \frac{3}{2-x}$ (1,0)

$\int dy = \int \frac{3}{2-x}\,dx$

$y = 3\int \frac{1}{2-x}\,dx$

$u = 2-x$
$du = -1\,dx$

$y = -3\int \frac{1}{u}\,du$

$y = -3\ln|2-x| + C$