5.2 The Natural Logarithmic Function: Integration

THEOREM 5.5 LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

#1

$$\int \frac{10}{x}\,dx = 10\int \frac{1}{x}\,dx$$

$$= 10\ln|x| + C$$

#2 $\int_0^1 \frac{1}{4-3x}\,dx$ $= \int_0^1 \frac{1}{(4-3x)^1}\,dx$

$u = 4-3x$

$du = -3\,dx$

$\frac{du}{-3} = dx$

$= \int_4^1 \frac{1}{u}\frac{du}{-3} = -\frac{1}{3}\int_4^1 \frac{1}{u}\,du = \frac{1}{3}\int_1^4 \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u|\Big|_1^4 = \frac{1}{3}\ln 4 - \frac{1}{3}\ln 1 = \frac{1}{3}\ln 4$

or

$\frac{2}{3}\ln 2$

$\ln\sqrt[3]{4}$

3.) $\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \frac{x}{u}\,\frac{du}{2x}$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

$= \ln\sqrt{x^2+1} + C$

4.) $\int \frac{\cos x}{1+\sin x}\,dx$

$u = 1+\sin x$

$du = \cos x\,dx$

$\int \frac{1}{u}\,du$

$\ln|u| + C$

$\ln|1+\sin x| + C$

or $\ln(1+\sin x) + C$

$$5) \int \frac{x-1}{x} dx = \int \left(\frac{x}{x} - \frac{1}{x}\right) dx$$

$$= \int \left(1 - \frac{1}{x}\right) dx = x - \ln|x| + C$$

Degree numerator $\geq$ degree denominator

Use division

$$y = \frac{1}{2} \ln|x^2+1|$$

$$y' = \frac{1}{2} \cdot \frac{2x}{x^2+1}$$

$$y' = \frac{x}{x^2+1}$$

#6 $$\int \frac{2x^2 + 7x - 3}{x - 2}\,dx = \int \left(2x + 11 + \frac{19}{x-2}\right)dx$$

$$2 \mid 2 \quad 7 \quad -3$$
$$\quad\quad 4 \quad 22$$
$$2 \quad 11 \quad 19$$

$$x^2 + 11x + 19\int \frac{1}{x-2}\,dx$$

$$x^2 + 11x + 19\int \frac{1}{u}\,du$$

$$x^2 + 11x + 19\ln|u| + C$$

$$x^2 + 11x + 19\ln|x-2| + C$$

7.) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} du$

$-\ln|u| + C$

$-\ln|\cos x| + C$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

#8: Solve the differential equation given the initial condition.

$$\frac{ds}{d\theta} = \tan 2\theta \qquad (0, 2)$$

$\theta \quad s$

$s = \int \tan 2\theta \, d\theta$

$u = 2\theta$

$du = 2d\theta$

$\frac{du}{2} = d\theta$

$s = \frac{1}{2}\int \tan u \, du$

$s = \frac{1}{2}(-\ln|\cos u|) + C$

$s = -\frac{1}{2}\ln|\cos 2\theta| + C$

$2 = -\frac{1}{2}\ln|\cos 0| + C$

$2 = C$

$\boxed{s = -\frac{1}{2}\ln|\cos 2\theta| + 2}$

INTEGRALS OF THE SIX BASIC TRIGONOMETRIC FUNCTIONS

$$\int \sin u\,du = -\cos u + C \qquad \int \cos u\,du = \sin u + C$$

$$\int \tan u\,du = -\ln|\cos u| + C \qquad \int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C \qquad \int \csc u\,du = -\ln|\csc u + \cot u| + C$$

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9.) $\int \sec\frac{x}{2}\,dx$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2du = dx$

$2\int \sec u\,du$

$2\ln\left|\sec\frac{x}{2} + \tan\frac{x}{2}\right| + C$