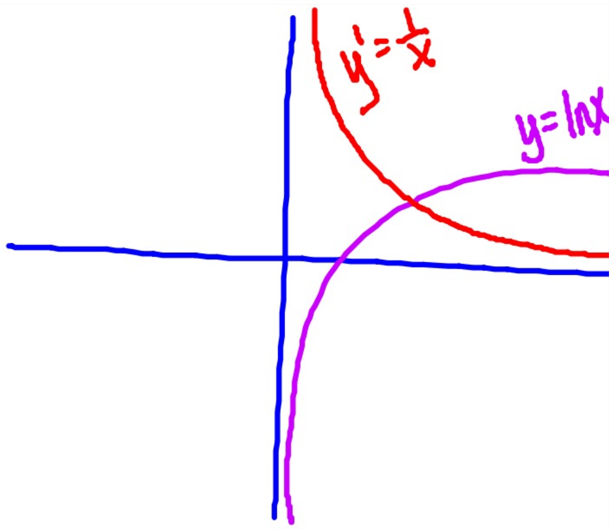
5.1 The Natural Logarithmic Function: Differentiation

- Develop and use properties of the natural logarithmic function.
- Understand the definition of the number e.
- Find derivatives of functions involving the natural logarithmic function.

Derivative of y = lnx

$$\frac{d}{dx}[\ln x] = \frac{1}{x} \quad x > 0$$



Derivative of y = ln u

$$\frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u} \quad u > 0$$

$$y = \ln(2x+1)$$

$$y' = \frac{1}{2x+1} \cdot 2$$

$$y' = \frac{2}{2x+1}$$

Find the derivative.

#1 $$h(t) = \frac{\ln t}{t}$$

$$h'(t) = \frac{t \cdot \frac{1}{t} - \ln t \cdot 1}{t^2}$$

$$h'(t) = \frac{1 - \ln t}{t^2}$$

#2 $$y = \ln \sqrt[3]{\frac{x-1}{x+1}}$$

$$y = \frac{1}{3}\left(\ln(x-1) - \ln(x+1)\right)$$

$$y' = \frac{1}{3}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

THEOREM 5.4 DERIVATIVE INVOLVING ABSOLUTE VALUE

If u is a differentiable function of x such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}.$$

proof
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#3

$$f(x) = \ln|\cos x|.$$

$$f'(x) = \frac{1}{\cos x} \cdot -\sin x$$

$$= \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

#4: Find the relative extrema. Justify. $D:\{x \mid x>0\}$

$$y = x - \ln x$$

$$y' = 1 - \frac{1}{x}$$

$$y' = \frac{x-1}{x}$$

$-$ $+$ y'

0 $\frac{1}{2}$ 1 2

rel min @ $(1, 1)$
because y' changes from neg. to pos. at this point.

#5: Find y' using log differentiation.

$$y = \sqrt{\frac{x^2-1}{x^2+1}}, \quad x > 1$$

#6: Find dy/dx using implicit differentiation.

$$4xy + \ln(x^2y) = 7$$

$$4xy + 2\ln x + \ln y = 7$$

$$4\left(x\frac{dy}{dx} + y\cdot 1\right) + \frac{2}{x}\cdot 1 + \frac{1}{y}\cdot\frac{dy}{dx} = 0$$

$$4x\frac{dy}{dx} + 4y + \frac{2}{x} + \frac{1}{y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\left(-4y - \frac{2}{x}\right)\cdot xy}{\left(4x + \frac{1}{y}\right)\cdot xy} = \frac{-4xy^2 - 2y}{4x^2y + x}$$

#7: Find the equation of the tangent line at the given point.

$f(x) = 3x^2 - \ln x, \quad x = 1$

$(1, 3)$

$m = 5$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 6 - 1$

$= 5$

$y - 3 = 5(x - 1)$

normal line

$y - 3 = \frac{-1}{5}(x - 1)$

THEOREM 5.5 LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$ 2. $\int \frac{1}{u}\,du = \ln|u| + C$

5.2 Notes

#1 $\int \frac{10}{x}\,dx$

$10\int \frac{1}{x}\,dx$

$10\ln|x| + C$

Check

$\frac{d}{dx}\left(10\ln|x| + C\right)$

$10 \cdot \frac{1}{x} + 0$

$\frac{10}{x}$ ✓

#2

$$\int \frac{1}{4-3x}\,dx = -\frac{1}{3}\int \frac{-3\cdot 1}{(4-3x)^1}\,dx$$

$$\int \frac{1}{x}dx = \ln|x| + c$$

$u = 4-3x$

$du = -3\,dx$

$$= -\frac{1}{3}\int \frac{1}{u}\,du$$

or

$$-\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ln|u| + C$$

$$= -\frac{1}{3}\ln|4-3x| + C$$

#3

$$\frac{1}{2}\int \frac{2\cdot x}{(x^2+1)^1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$$

$u = x^2+1$

$du = 2x\,dx$

$$\frac{1}{2}\ln|u| + C$$

$$\frac{1}{2}\ln|x^2+1| + C$$

Check: $\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}x}{x^2+1} + 0 = \frac{x}{x^2+1}$

#4 $\int \frac{2x^2+7x-3}{x-2}\,dx = \int \left(2x+11+\frac{19}{(x-2)}\right)dx$

degree num. $\geq$ degree den. division

$$\begin{array}{r} 2x+11 \\ x-2\,\overline{)\,2x^2+7x-3} \\ -\,2x^2+4x \\ \hline 11x-3 \\ -\,11x+22 \\ \hline 19 \end{array}$$

$x^2+11x+19\ln|x-2|+C$

#4 $\displaystyle\int \frac{x^3-3x^2+4x-9}{x^2+3}\,dx$

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2+3 & x^3-3x^2+4x-9 \\
 & -x^3 \quad \mp 3x \\
\hline
 & -3x^2+x-9 \\
 & +3x^2 \quad +9 \\
\hline
 & x
\end{array}
$$

$$\Rightarrow \int\left(x-3+\frac{x}{x^2+3}\right)dx$$

$u=x^2+3$

$du=2x\,dx$

$$\frac{x^2}{2}-3x+\frac{1}{2}\int\frac{2x}{x^2+3}\,dx$$

$$\frac{x^2}{2}-3x+\frac{1}{2}\ln\left(x^2+3\right)+C$$

#5 $$\int_e^{e^2} \frac{1}{x \ln x}\,dx = \int_e^{e^2} \frac{1}{x} \cdot \frac{1}{\ln x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

lower : $u(e) = 1$

upper : $u(e^2) = 2$

$$\int_1^2 \frac{1}{u}\,du = \ln|u| \Big]_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

#6 $-1\int_{0}^{\pi/3} \frac{-1\sin x}{(1+\cos x)}\,dx = -\int_{2}^{3/2} \frac{1}{u}\,du$

$u = 1+\cos x$

$du = -\sin x\,dx$

Lower: $u(0) = 2$

Upper: $u\left(\frac{\pi}{3}\right) = \frac{3}{2}$

$= \int_{3/2}^{2} \frac{1}{u}\,du$

$= \ln|u| \Big]_{3/2}^{2}$

$-\ln\frac{3}{4}$

$\ln\left(\frac{3}{4}\right)^{-1} = \ln\frac{4}{3}$

$= \ln 2 - \ln\frac{3}{2} = \ln\frac{2}{\frac{3}{2}}$

$= \ln\frac{4}{3}$