4.5 Change of Variable

$$\int_0^1 (3x-2)^3\,dx$$

$u = 3x - 2$

$du = 3\,dx$

Change the limits and evaluate the definite integral.

lower

$x = 0 \rightarrow u = -2$

upper

$x = 1 \rightarrow u = 1$

$\frac{du}{3} = dx$

$$\int_{-2}^{1} u^3 \frac{du}{3}$$

$$\frac{1}{3}\int_{-2}^{1} u^3\,du = \frac{1}{12}u^4\Big|_{-2}^{1}$$

$$= \frac{1}{12} - \frac{16}{12} = \frac{-15}{12}$$

ıg the substitution $u = 2x + 1$, $\int_0^2 \sqrt{2x+1}\,dx$ is equivalent to

$\frac{1}{2}\int_{-1/2}^{1/2} \sqrt{u}\,du$ (B) $\frac{1}{2}\int_0^2 \sqrt{u}\,du$ (C) $\frac{1}{2}\int_1^5 \sqrt{u}\,du$ (D) $\int_0^2 \sqrt{u}\,du$ (E) $\int_1^5 \sqrt{u}\,d$

$u = 2x+1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int_{-1}^{1} \frac{1}{2x+3}\,dx$$

Rewrite the definite integral in terms of u.

$u = 2x + 3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int_{1}^{5} \frac{1}{u}\,\frac{du}{2}$$

$$\frac{1}{2}\int_{1}^{5} \frac{1}{u}\,du$$

$$\int_0^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\,dx$$

Rewrite the definite integral in terms of u.

$u = 4 - x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$$\int_0^1 \frac{x}{\sqrt{u}} \frac{du}{-2x}$$

$$-\frac{1}{2}\int_4^1 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_1^4 u^{-1/2}\,du$$

77) $\int_1^2 2x^2\sqrt{x^3+1}\,dx$

$u = x^3 + 1$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$$\int_2^9 2x^2\sqrt{u}\,\frac{du}{3x^2} = \frac{2}{3}\int_2^9 u^{1/2}\,du$$

$$= \frac{2}{3}\cdot\frac{u^{3/2}}{3/2}\Big|_2^9$$

$$= \frac{4}{9}\left(9^{3/2} - 2^{3/2}\right)$$

$$\int x\sqrt{x+3}\,dx$$

$u = x+3 \rightarrow x = u-3$

$du = 1\,dx$

#67

$$\int (u-3)u^{1/2}\,du$$

$$\int u^{3/2} - 3u^{1/2}\,du = \frac{2}{5}u^{5/2} - \frac{2\cancel{3}u^{3/2}}{\cancel{3}} + C$$

$$= \frac{2}{5}(x+3)^{5/2} - \frac{10}{5}(x+3)^{3/2} + C$$

$$= \frac{2}{5}(x+3)^{3/2}\left(x+3-5\right) + C$$

$$= \frac{2}{5}(x+3)^{3/2}(x-2) + C$$

(1) $\int \frac{3}{(1-7x)^4}\,dx$

$\frac{1}{7}(1-7x)^{-3}+C$

(2) $\int_0^2 \sqrt{3x+10}\,dx$

$\frac{2}{9}(64-10^{3/2})$

(3) $\int \sin^4 x\cos x\,dx$

$\frac{\sin^5 x}{5}+C$

(4) $\int \sin\left(\frac{x}{10}\right)dx$

$10\int \sin u\,du$

$-10\cos\left(\frac{x}{10}\right)+C$

$u=\frac{x}{10}$

$du=\frac{1}{10}dx$

$10\,du=dx$