4.5 Integration with Substitution Part 2

#1

$$\left(-\frac{1}{2}\right)(-2)\int x\sqrt{1-x^2}\,dx$$

$$u = 1-x^2$$

$$du = -2x\,dx$$

$$-\frac{1}{2}\int \sqrt{u}\,du$$

$$-\frac{1}{2}\frac{u^{3/2}}{3/2} + C$$

$$-\frac{1}{3}(1-x^2)^{3/2} + C$$

#2

$$\int x\sin(x^2)\,dx$$

$$u = x^2 \quad du = 2x\,dx$$

$$\frac{du}{2x} = dx$$

$$\int \cancel{x}\sin u\,\frac{du}{2\cancel{x}}$$

$$\frac{1}{2}\int \sin u\,du$$

$$-\frac{1}{2}\cos x^2 + C$$

#3 $\int_0^{\pi/6} \cos 3x (dx)$

$u = 3x \quad du = 3dx$

$\frac{du}{3} = dx$

$\frac{1}{3}\int \cos u \, du$

$\frac{1}{3}\sin u + C = \frac{1}{3}\sin 3x \Big|_0^{\pi/6}$

$\boxed{\frac{1}{3}}$

#3 $\int_0^{\pi/6} \cos 3x \, dx$

change of variable (only for definite integrals

$u = 3x \qquad du = 3\,dx$

Lower

$x = 0 \rightarrow u = 0$

upper

$x = \frac{\pi}{6} \rightarrow u = \frac{\pi}{2}$

$\frac{1}{3}\int_0^{\pi/2} \cos u \, du$

$\frac{1}{3} \sin u \Big|_0^{\pi/2}$

$\frac{1}{3}(1-0)$

$\frac{1}{3}$

$$\int_0^1 x(x^2+1)^3\,dx$$

$u = x^2+1$

$du = 2x\,dx$

<u>lower</u>

$x=0 \rightarrow u=1$

<u>upper</u>

$x=1 \rightarrow u=2$

$$\frac{1}{2}\int_1^2 u^3\,du$$

$$\frac{1}{2}\cdot\frac{u^4}{4}\Big|_1^2$$

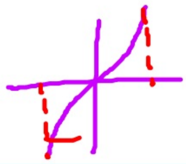
$$\frac{16}{8}-\frac{1}{8}=\frac{15}{8}$$

THEOREM 4.16 INTEGRATION OF EVEN AND ODD FUNCTIONS

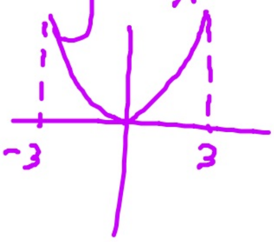
Let f be integrable on the closed interval $[-a, a]$.

1. If f is an *even* function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

2. If f is an *odd* function, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$.



$y = x^2$



$$\int_{-3}^{3} x^2\,dx = 2\int_{0}^{3} x^2\,dx$$

$$\int_{-\pi/2}^{\pi/2} \sin x \cos x \, dx = 0$$

Skip 83

$u = \sin x \quad du = \cos x \, dx$

Even/odd ?

$\sin(-x)\cos(-x)$

$-\sin x \cos x$

odd

$$\int u \, du = \frac{u^2}{2}$$

$$= \frac{\sin^2 x}{2} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2} - \frac{1}{2} = 0$$

33.) $\int x^{-1/2}(x^2+5x-8)\,dx$

$= \int \left(x^{3/2}+5x^{1/2}-8x^{-1/2}\right)dx$

29.) $\int \left(1+\frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt = -\int u^3 \, du$

$u = 1 + \frac{1}{t}$

$du = -\frac{1}{t^2} dt$

$-t^2 du = dt$

$-\frac{u^4}{4} + C$

$-\frac{1}{4}\left(1+\frac{1}{t}\right)^4 + C$

51.) $\int \frac{1}{\theta^2} \cos\frac{1}{\theta}\, d\theta$

$u = \frac{1}{\theta}$

$du = -\frac{1}{\theta^2} d\theta$

$-\theta^2 du = d\theta$

$\int \frac{1}{\cancel{\theta^2}} \cos u \left(-\cancel{\theta^2} du\right)$

$-\int \cos u\, du = -\sin u + C$

$= -\sin\frac{1}{\theta} + C$

55.) $\int \tan^4 x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \int u^4 \, du$