

You will need to start bring your graphing calculator every day starting next Monday, January 11th.

Tonight's HW: Skip #67

## 4.5 Integration by Substitution

Sometimes, you will encounter a more complicated integrand. We can use u-substitution when there is a composite function. (think backwards chain rule)

The diagram illustrates the integration by substitution formula. It features three pink-bordered boxes with arrows pointing to parts of the equation  $\int f(g(x))g'(x) dx = F(g(x)) + C$ . The top box, labeled "Outside function", has an arrow pointing to the  $f(g(x))$  term. The bottom-left box, labeled "Inside function", has an arrow pointing to the  $g(x)$  term. The bottom-right box, labeled "Derivative of inside function", has an arrow pointing to the  $g'(x)$  term, which is also indicated by a pink curly brace.

$$\int f(g(x))g'(x) dx = F(g(x)) + C$$

#1

$$\int (x^2 - 9)^3 (2x) dx$$

$$u = x^2 - 9$$
$$du = 2x dx$$

$$\int u^3 du = \frac{1}{4} u^4 + C$$

$$= \frac{1}{4} (x^2 - 9)^4 + C$$

---

check:  $(x^2 - 9)^3 \cdot 2x$

#2

$$\frac{1}{3} \cdot 3 \int x^2 (x^3 + 5)^4 dx$$

$$u = x^3 + 5$$
$$du = 3x^2 dx$$

$$\frac{1}{3} \int u^4 du = \frac{1}{3} \cdot \frac{1}{5} \cdot u^5 + C$$

$$= \frac{1}{15} (x^3 + 5)^5 + C$$

$$\int \frac{1}{u^s} du$$

$$\int u^{-s} du$$

#3

$$\frac{1}{4} \cdot 4 \int \frac{x^3}{\sqrt{1+x^4}} dx$$

$$u = 1 + x^4$$
$$du = 4x^3 dx$$

$$\frac{1}{4} \int \frac{du}{\sqrt{u}} = \frac{1}{4} \int \frac{1}{\sqrt{u}} du$$

$$= \frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \cdot \frac{u^{1/2}}{1/2} + C$$

$$\frac{1}{2} \sqrt{1+x^4} + C$$

#3

$$\int \frac{x^3}{\sqrt{1+x^4}} (dx)$$

$$\int \frac{\cancel{x^3}}{\sqrt{1+x^4}} \frac{du}{4\cancel{x^3}}$$

$$\frac{1}{4} \int u^{-1/2} du$$

$$u = 1 + x^4$$
$$du = 4x^3 dx$$
$$\frac{du}{4x^3} = dx$$

#4

$$\int_0^2 \frac{x}{\sqrt{1+2x^2}} dx$$

$$u = 1 + 2x^2$$
$$du = 4x dx$$
$$\frac{du}{4x} = dx$$

$$\int \frac{\cancel{x}}{\sqrt{1+2x^2}} \cdot \frac{du}{\cancel{4x}}$$

$$\frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \cdot \frac{u^{1/2}}{1/2} = \frac{1}{2} \sqrt{1+2x^2} \Big|_0^2$$
$$= 1$$

#5

$$\int \frac{t - 9t^2}{\sqrt{t}} dt$$

Not EVERY question  
will need u-sub!!!!

$$\int t^{-1/2} (t - 9t^2) dt$$

$$\int (t^{1/2} - 9t^{3/2}) dt$$

$$\frac{2}{3} t^{3/2} - \frac{18}{5} t^{5/2} + C$$



$$\#6 \int \sin 7x dx$$
$$\int \sin u \frac{du}{7}$$

$$u = 7x$$
$$du = 7 dx$$
$$\frac{du}{7} = dx$$

$$\frac{1}{7} \int \sin u du = -\frac{1}{7} \cos u + C$$
$$= -\frac{1}{7} \cos 7x + C$$

$$\#7 \quad \int \tan^2 x \, dx$$

$$1 + \tan^2 x = \sec^2 x$$

$$\int (\sec^2 x - 1) \, dx$$

$$\tan x - x + C$$

$$\#8) \int \cot^3 x \csc^2 x \, dx$$

$$\int (\cot x)^3 \csc^2 x \, dx$$

$$\begin{aligned} u &= \cot x \\ du &= -\csc^2 x \, dx \\ \frac{du}{-\csc^2 x} &= dx \end{aligned}$$

$$-\int u^3 \, du$$

$$= -\frac{u^4}{4} + C = -\frac{1}{4} \cot^4 x + C$$