REM 4.11 THE SECOND FUNDAMENTAL THEOREM OF CALCULUS

continuous on an open interval I containing a, then, for every x in the
l,

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x).$$

$F(x) = \int_2^x \tan\theta \, d\theta$

$F'(x) = \tan x$

FIND F(X) AND F'(X)

$F'(x) = \sin x$

#1

$$F(x) = \int_0^x \sin\theta \, d\theta$$

$F(x) = -\cos\theta \Big]_0^x$

$= -\cos x - (-\cos 0)$

$F(x) = -\cos x + 1$

#2

$$F(x) = \int_0^x (4t - 7)\, dt$$

$$F'(x) = 4x - 7$$

#3 $F(x) = \int_1^{x^3} t^2\,dt$

$F'(x) = x^6 \cdot 3x^2$

$F(x) = \left.\frac{t^3}{3}\right]_1^{x^3}$

$F(x) = \frac{x^9}{3} - \frac{1}{3}$

$F'(x) = 3x^8$

4 $F(\theta) = \int_{1}^{\sin\theta} \sqrt{t^2+6}\,dt$

$F'(\theta) = \sqrt{\sin^2\theta + 6} \cdot \cos\theta$

$F'(\pi) = \sqrt{0+6} \cdot (-1)$

$= -\sqrt{6}$

5 $f(x) = \int_{1}^{x^2} t\sqrt{t+1}\,dt$

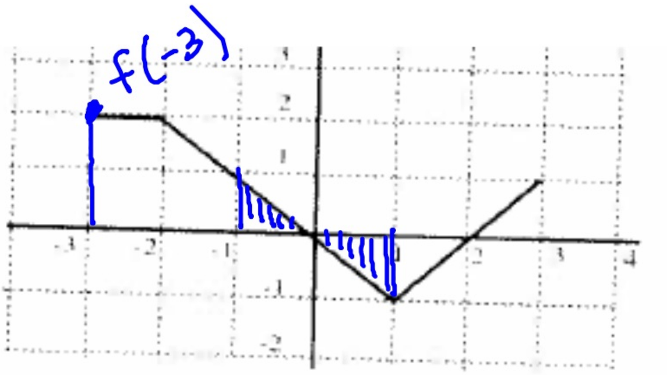
$f'(x) = x^2\sqrt{x^2+1} \cdot 2x$

$f'(2) = 16\sqrt{5}$

Accumulation Functions
(2nd Fundamental Theorem)

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

1) Find g(-3) and g'(-3)



This is the graph of f

$$g(-3) = \int_1^{-3} f(t)\,dt$$

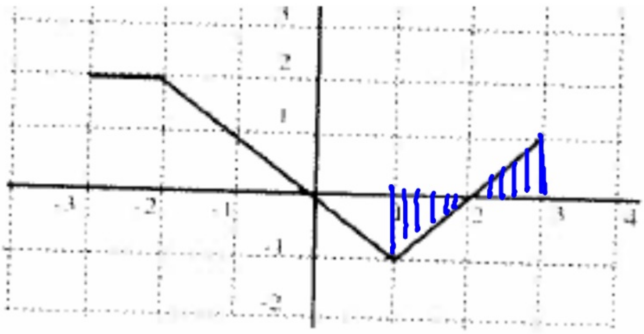
$$g(-3) = -\int_{-3}^{1} f(t)\,dt = -3.5$$

$$g'(x) = f(x)$$

$$g'(-3) = f(-3) = 2$$

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

2) Find g(3) and g'(3)



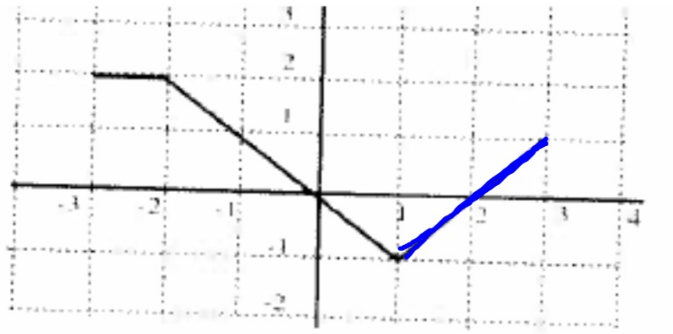
This is the graph of f

$g(3) = \int_1^3 f(t)\,dt = 0$

$g'(x) = f(x)$

$g'(3) = f(3) = 1$

Let g be defined as $g(x) = \int_1^x f(t)\,dt$


This is the graph of f

3) Find g'(2) and g''(2)

$g'(x) = f(x)$

$g'(2) = f(2) = 0$

$g''(x) = f'(x)$

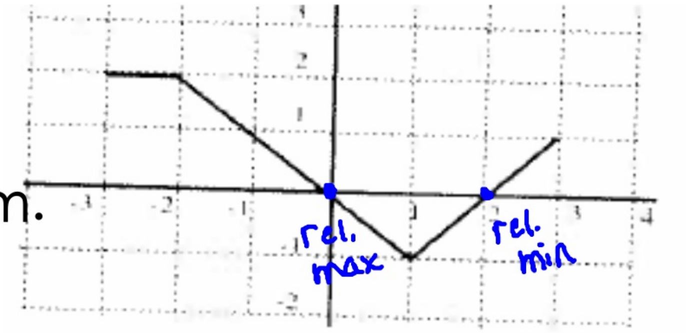
$g''(2) = f'(2) = 1$

slope of f(x) at x=2

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

4) Find all values of x on the open interval (-3, 3) at which g attains a relative maximum. Justify

$g'(x) = f(x)$



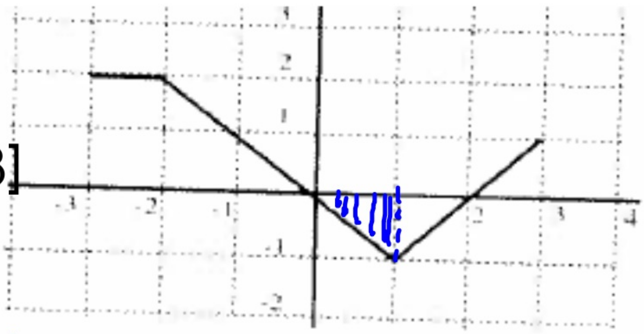
This is the graph of f

$g'(x)$

Since g'(x) = f(x), there is a relative maximum at x = 0 because g'(x) changes from positive to negative at this point.

Let g be defined as $g(x)=\int_1^x f(t)\,dt$

5) Find the absolute maximum value of g on the closed interval [-3,3]



This is the graph of f

check endpoints and critical points (rel max)

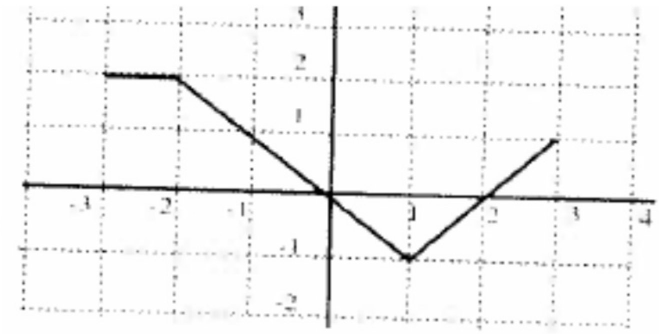
x	g(x)
-3	-3.5
0	0.5
3	0

Absolute maximum value: 0.5

$$g(0)=\int_1^0 f(t)\,dt = -\int_0^1 f(t)\,dt$$

Let g be defined as $g(x) = \int_1^x f(t)\,dt$

6) Find all values of x in the open interval (-3,3) at which the graph of g has a point of inflection.



This is the graph of f

$g'(x) = f(x)$

Point of inflection at x = 1 because g'(x) = f(x)
and the slope of g'(x) changes signs
at this point.

$g''(x) = f'(x)$

Point of inflection at x = 1
because g''(x) = f'(x) and
g''(x) changes signs
at x = 1