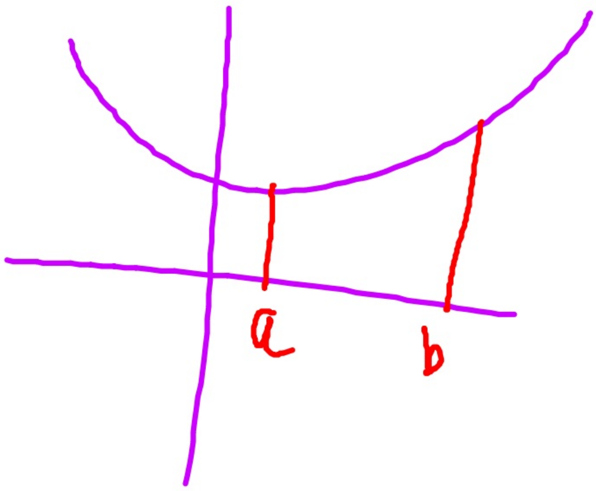
4.4 The Fundamental Theorem of Calculus

How we calculate the exact value of a definite integral.

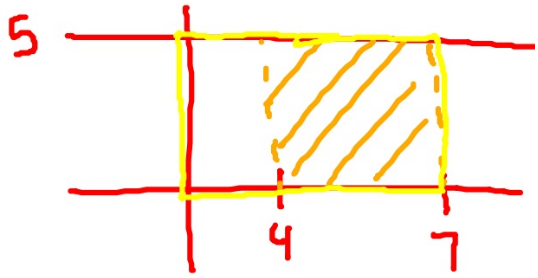


THEOREM 4.9 THE FUNDAMENTAL THEOREM OF CALCULUS

If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Evaluate the definite integral.



#1 $\int_4^7 5\,dv = 5v$

$5(7) - 5(4)$

$35 - 20$

15

#2 $\int_1^2 (6x^2 + 2x - 3)\,dx = 2x^3 + x^2 - 3x + C \Big|_1^2$

$(16 + 4 - 6 + C) - (2 + 1 - 3 + C)$

$14 + C - C$

14

#3 $$\int_0^2 (2-t)\sqrt{t}\,dt$$

$$\int_0^2 \left(2t^{1/2} - t^{3/2}\right)dt$$

$$2\cdot\frac{t^{3/2}}{3/2} - \frac{t^{5/2}}{\frac{5}{2}} + c = \frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2} + c\Big|_0^2$$

$$= \frac{4}{3}\cdot 2^{3/2} - \frac{2}{5}\cdot 2^{5/2}$$

#4 $\int_0^4 |x-1|\,dx$

DO NOT USE CALCULUS!!
SKETCH AND FIND THE
ANSWER GEOMETRICALLY!

$$\int_0^1 (-x+1)\,dx + \int_1^4 (x-1)\,dx$$

$$\frac{1}{2}(1)(1) + \frac{1}{2}(3)(3)$$

$$\frac{1}{2} + \frac{9}{2} = 5$$

DEFINITION OF THE AVERAGE VALUE OF A FUNCTION ON AN INTERVAL

If f is integrable on the closed interval $[a, b]$, then the **average value** of f on the interval is

$$\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Finding an average means finding the value of the definite integral and dividing by the width of the interval.

$$\int_4^1 f(x)\,dx = -\int_1^4 f(x)\,dx$$

#6 Find the average value.

$f(x) = \cos x, \quad [0, \pi/2]$

$$\frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} \cos x \, dx = \frac{2}{\pi} (\sin x + C) \Big|_0^{\pi/2}$$

$$\frac{2}{\pi} \left(\left(\sin \frac{\pi}{2} + C\right) - (\sin 0 + C) \right)$$

$$\frac{2}{\pi}$$