4.3 Riemann Sums and Definite Integrals

$$\int f'(x)\,dx = f(x) + C$$

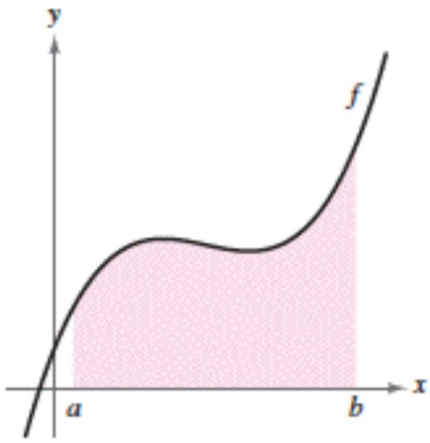
indefinite integral

THEOREM 4.5 THE DEFINITE INTEGRAL AS THE AREA OF A REGION

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx.$$

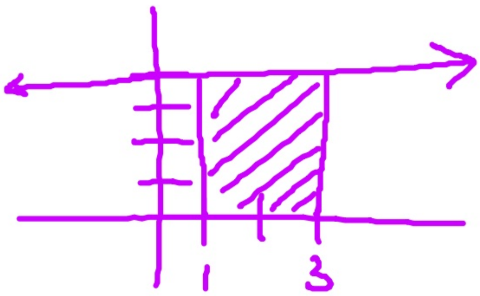
numerical answer



Sketch, then find the value of the definite integral.

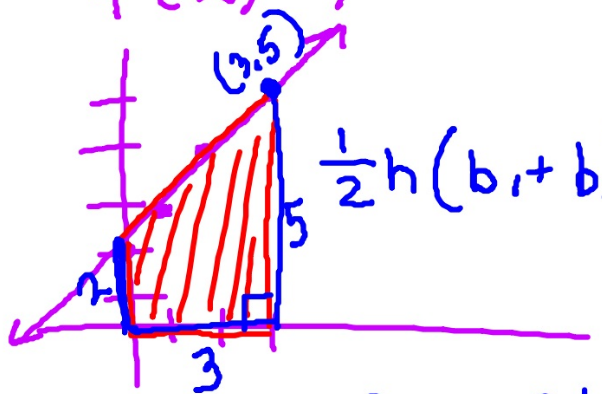
#1 $\int_1^3 4\,dx = 8$

$f(x) = 4$



#2 $\int_0^3 (x+2)\,dx = \frac{21}{2}$

$f(x) = x + 2$

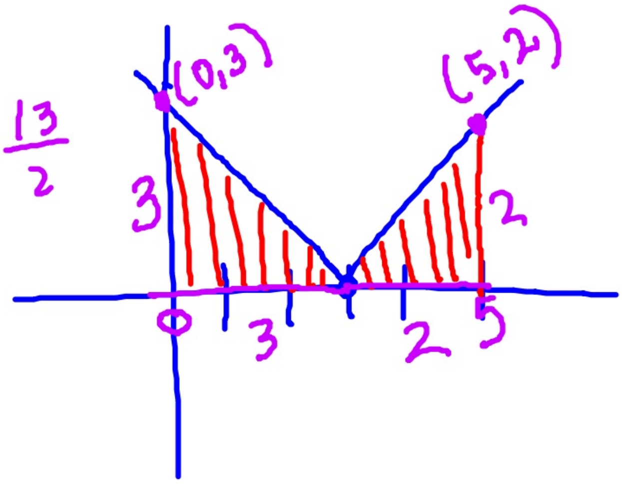


$\frac{1}{2}h(b_1 + b_2)$

$\frac{1}{2} \cdot 3(2+5) = \frac{21}{2}$

#3

$$\int_0^5 |x-3|\,dx = \frac{13}{2}$$



$$\frac{1}{2}(3)(3) + \frac{1}{2}(2)(2)$$

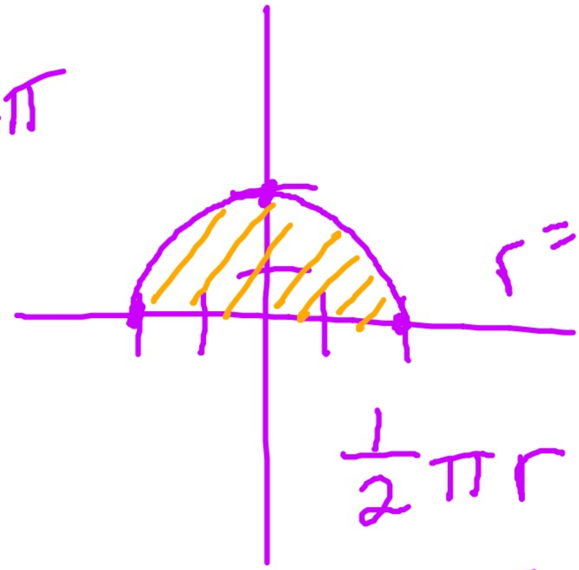
$$\frac{9}{2} + \frac{4}{2} = \frac{13}{2}$$

#4 $\int_{-2}^{2} \sqrt{4 - x^2}\, dx = 2\pi$

$f(x) = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$



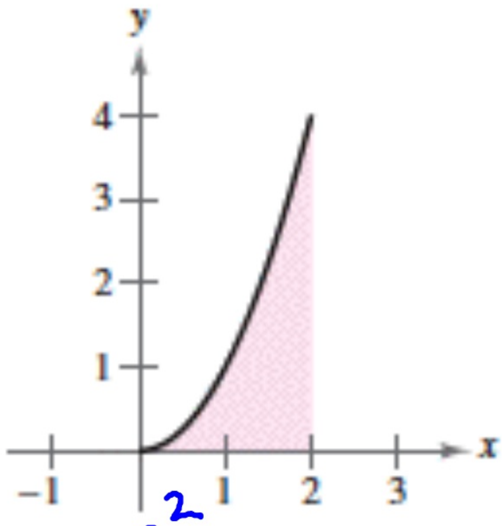
$r = 2$

$\frac{1}{2}\pi r^2$

2π

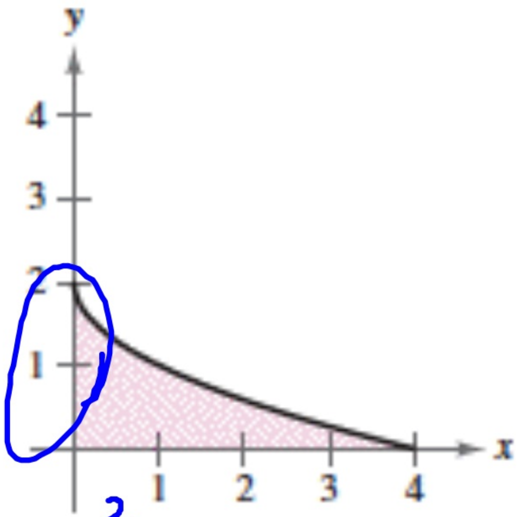
Set up a definite integral that would represent the area of the region. Do not evaluate.

#5 $f(x) = x^2$



$$\int_0^2 x^2\,dx$$

#6 $f(y) = (y-2)^2$



$$\int_0^2 (y-2)^2\,dy$$

4.3 continued

DEFINITIONS OF TWO SPECIAL DEFINITE INTEGRALS

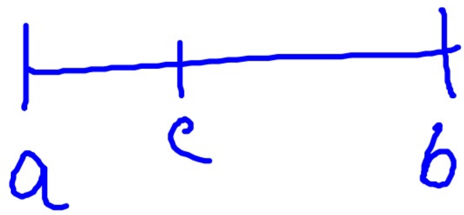
1. If f is defined at $x = a$, then we define $\int_a^a f(x)\,dx = 0$.

2. If f is integrable on $[a, b]$, then we define $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$.

THEOREM 4.6 ADDITIVE INTERVAL PROPERTY

If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$



THEOREM 4.7 PROPERTIES OF DEFINITE INTEGRALS

If f and g are integrable on $[a, b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a, b]$, and

1. $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$

#1

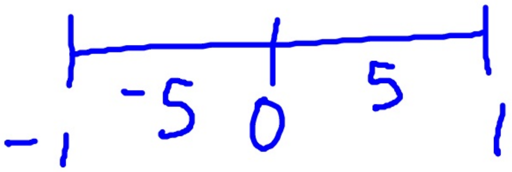
Given $\int_{-1}^{1} f(x)\,dx = 0$ and $\int_{0}^{1} f(x)\,dx = 5$, evaluate

(a) $\int_{-1}^{0} f(x)\,dx. = -5$

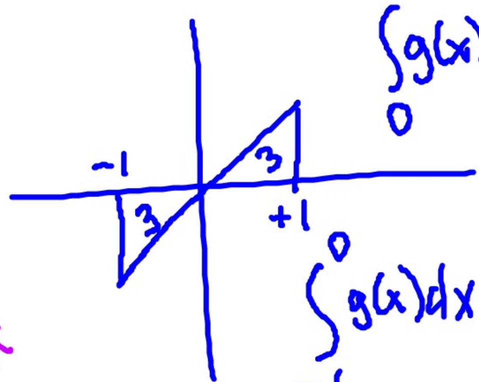
(b) $\underbrace{\int_{0}^{1} f(x)\,dx}_{5} - \underbrace{\int_{-1}^{0} f(x)\,dx}_{-5}. = 10$

(c) $\int_{-1}^{1} 3f(x)\,dx.$ 0

(d) $\int_{0}^{1} 3f(x)\,dx. = 15$



$$\int_{1}^{0} g(x)\,dx = -\int_{0}^{1} g(x)\,dx$$



$$\int_{0}^{-1} g(x)\,dx = +3$$

$$\int_{-1}^{0} g(x)\,dx = -3$$

$$\int_{-1}^{1} g(x)\,dx = 0$$

#2 Evaluate $\int_1^3 (-x^2 + 4x - 3)\,dx$ using each of the following values.

$$\int_1^3 x^2\,dx = \frac{26}{3}, \qquad \int_1^3 x\,dx = 4, \qquad \int_1^3 dx = 2$$

$$\int_1^3 -x^2\,dx + \int_1^3 4x\,dx - \int_1^3 3\,dx$$

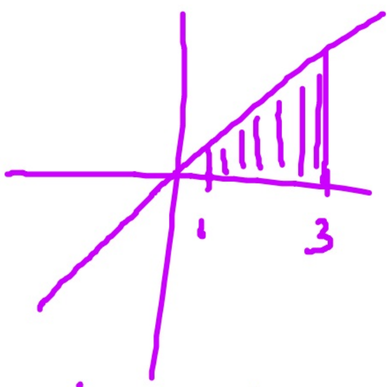
$$-\int_1^3 x^2\,dx + 4\int_1^3 x\,dx - 3\int_1^3 dx$$

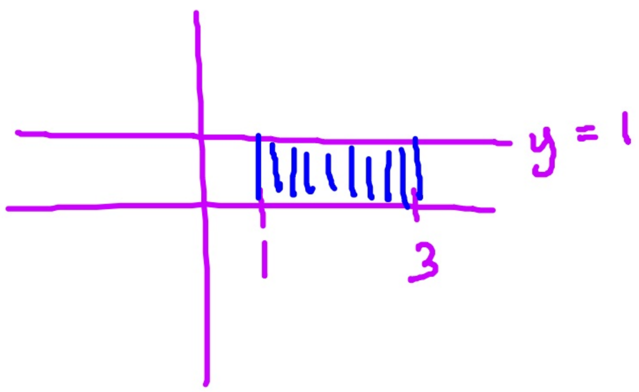
$$\frac{-26}{3} + 16 - 6$$

$$-8\tfrac{2}{3} + 10 = 1\tfrac{1}{3}$$

$$\int_1^3 x\,dx$$

$$\int_1^3 1\,dx = 2$$

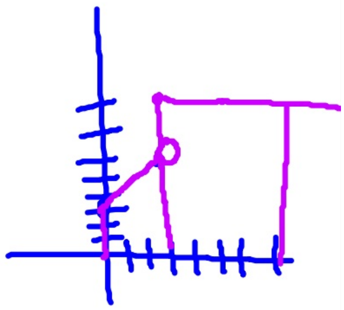




$$\frac{1}{2}\cdot 2(1+3)$$

$$4$$

$$g(x) = \begin{cases} x+3, & x < 3 \\ 8 & x \geq 3 \end{cases}$$



$$\int_0^7 g(x)\,dx = \int_0^3 (x+3)\,dx + \int_3^7 8\,dx$$

$$\tfrac{1}{2}h(b_1+b_2) \quad + \quad 32$$

$$\tfrac{1}{2}(3)(3+6)$$

$$\frac{27}{2} + 32 = 45\tfrac{1}{2}$$