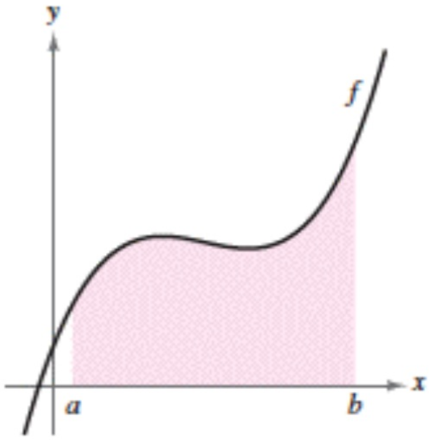
4.3 Riemann Sums and Definite Integrals

THEOREM 4.5 THE DEFINITE INTEGRAL AS THE AREA OF A REGION

If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx.$$

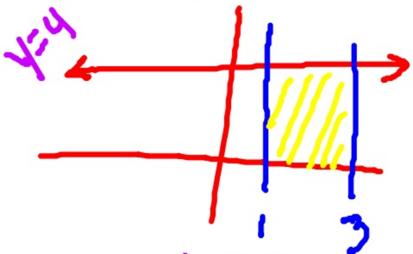


Sketch, then find the value of the definite integral.

#1 $\int_1^3 4\,dx$

$\boxed{8}$

$f(x) = 4$



L·w

2·4

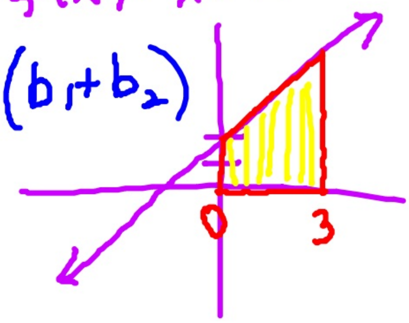
#2 $\int_0^3 (x + 2)\,dx$

$\frac{1}{2}(3)(2 + 5)$

$\boxed{21/2}$

$f(x) = x + 2$

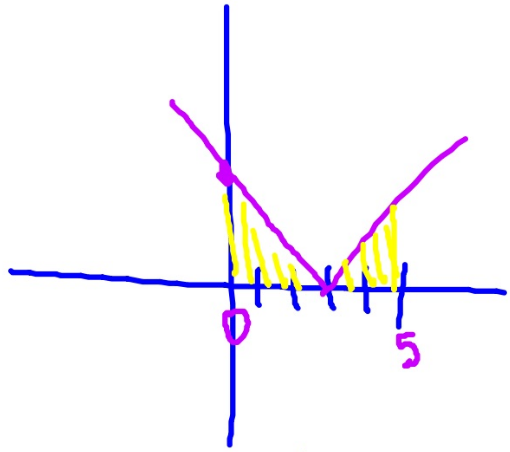
trap $= \frac{1}{2}h(b_1 + b_2)$



#3

$$\int_0^5 |x-3|\,dx$$

$$\boxed{\frac{13}{2}}$$



$\frac{1}{2}bh + \frac{1}{2}bh$

$\frac{1}{2}(3)(3) + \frac{1}{2}(2)(2)$

$\frac{9}{2} + 2$

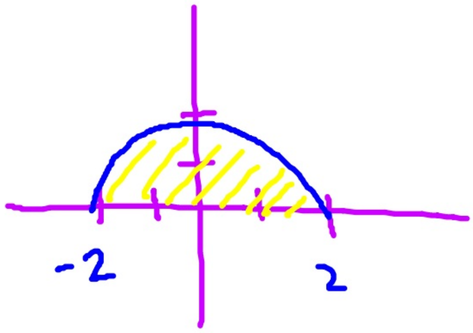
#4 $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$

*

$\frac{1}{2}\pi r^2$

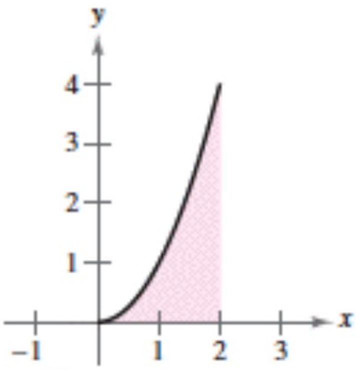
$\frac{1}{2}\pi (2)^2$

2π



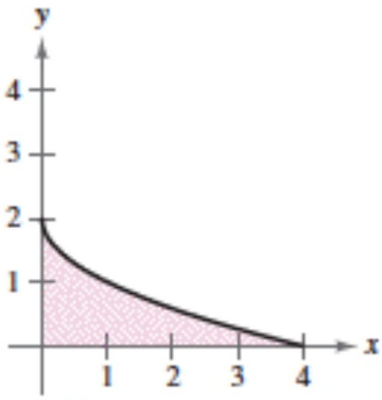
Set up a definite integral that would represent the area of the region. Do not evaluate.

#5 $f(x) = x^2$



$\int_0^2 x^2\,dx$

#6 $f(y) = (y-2)^2$



$\int_0^2 (y-2)^2\,dy$

DEFINITIONS OF TWO SPECIAL DEFINITE INTEGRALS

1. If f is defined at $x = a$, then we define $\displaystyle\int_a^a f(x)\,dx = 0$.

2. If f is integrable on $[a, b]$, then we define $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$.

THEOREM 4.6 ADDITIVE INTERVAL PROPERTY

If f is integrable on the three closed intervals determined by a, b, and c, then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

THEOREM 4.7 PROPERTIES OF DEFINITE INTEGRALS

If f and g are integrable on $[a, b]$ and k is a constant, then the functions kf and $f \pm g$ are integrable on $[a, b]$, and

1. $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

2. $\displaystyle\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$

#1 Given $\int_{-1}^{1} f(x)\,dx = 0$ and $\int_{0}^{1} f(x)\,dx = 5$, evaluate

(a) $\int_{-1}^{0} f(x)\,dx. = -5$

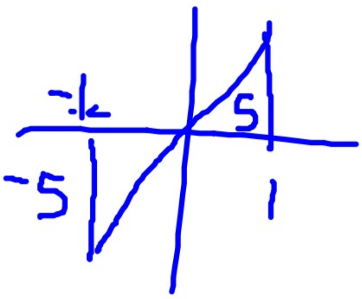
(b) $\int_{0}^{1} f(x)\,dx - \int_{-1}^{0} f(x)\,dx.$

(c) $\int_{-1}^{1} 3f(x)\,dx. = 0$

$3\int_{-1}^{1} f(x)\,dx$

(d) $\int_{0}^{1} 3f(x)\,dx. = 3\int_{0}^{1} f(x)\,dx$

$= 15$



$5 - (-5) = 10$

#2 Evaluate $\int_1^3 (-x^2 + 4x - 3)\,dx$ using each of the following values.

$$\int_1^3 x^2\,dx = \frac{26}{3}, \qquad \int_1^3 x\,dx = 4, \qquad \int_1^3 1\,dx = 2$$

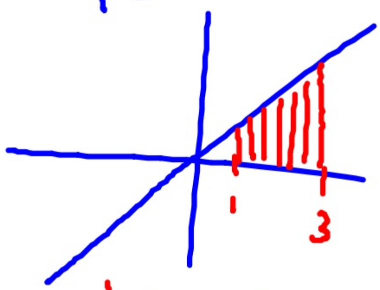
$$\int_1^3 -x^2\,dx + \int_1^3 4x\,dx - \int_1^3 3\,dx$$

$$-\int_1^3 x^2\,dx + 4\int_1^3 x\,dx - 3\int_1^3 dx$$

$$-\frac{26}{3} + 16 - 6$$

$$\int_1^3 x\,dx = 4$$

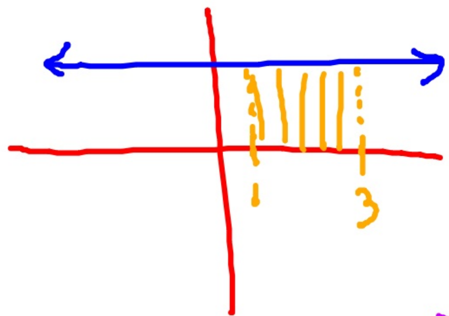
$$f(x) = x$$



$$\frac{1}{2}(2)(1+3)$$

$$\int_1^3 dx = 2$$

$$f(x) = 1$$



$$(2)(1)$$

84) $a(t) = \cos t$ $v(0) = 0$

$x(0) = 3$

$v(t) = \sin t + C$

$0 = C$

$v(t) = \sin t$

$x(t) = -\cos t + C$

$C = 4$

$x(t) = -\cos t + 4$

$$\int f(x)\,dx = F(x) + C$$

$$\int f(x)\,dx = G(x) + C$$

$$\int (x+1)(2x-3)\,dx = \int (2x^2 - x - 3)\,dx$$

$$= \frac{2x^3}{3} - \frac{x^2}{2} - 3x + C$$

$$\int \frac{x^2 - 5x + 7}{x^7}\,dx = \int (x^{-5} - 5x^{-6} + 7x^{-7})\,dx$$

$$\frac{-x^{-4}}{4} + \frac{5x^{-5}}{5} - \frac{7x^{-6}}{6} + C$$