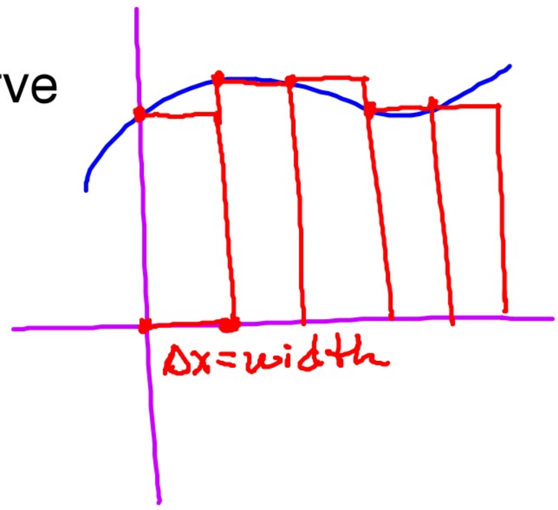
Left, Right, Midpoint and Trapezoidal Sums

Approximating area under a curve



If there is a constant width, the width can be calculated by:

Width = (b - a)/n

$[0, 12]$ $n = 4$ rectangles

$\frac{12-0}{4} = 3 =$ width

Left sum: Start at the LEFT of the interval

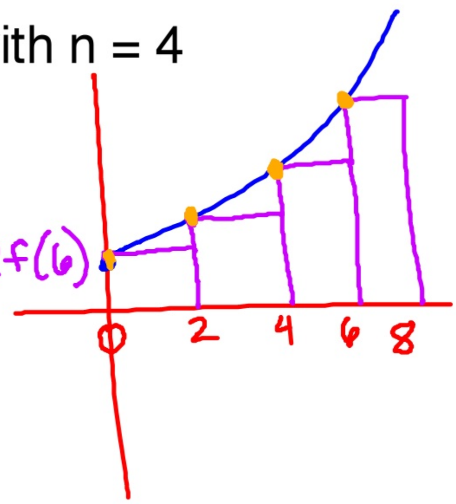
f(x) = x^2 + 1 on the interval [0, 8] with n = 4

$w = \frac{8-0}{4} = 2$

$$\int_0^8 f(x)\,dx \approx 2\cdot f(0) + 2\cdot f(2) + 2f(4) + 2f(6)$$

$$\approx 2[1 + 5 + 17 + 37)$$

$$\approx 120$$



$n \to \infty$

$w \to 0$

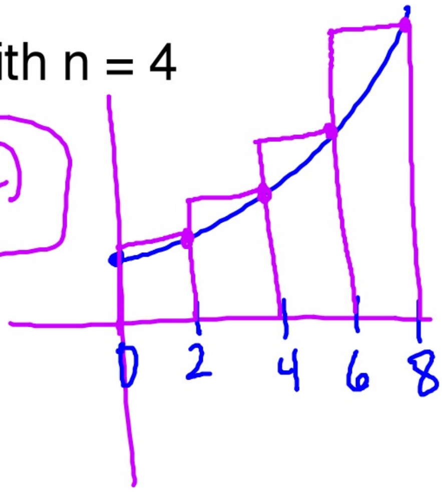
Right Sum: start at the right of the interval

$f(x) = x^2 + 1$ for the interval [0, 8] with n = 4

$$\int_0^8 f(x)\,dx \approx 2\left[f(8)+f(6)+f(4)+f(2)\right]$$

$$2\left[65+37+17+5\right]$$

$$\approx 248$$



Right sum with a chart (5 sub-intervals) g(x) is diff.

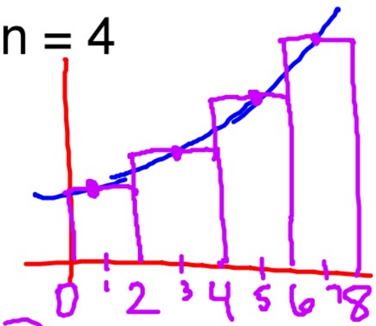
X	0	2	3	7	12	13
g(x)	1	4	8	5	8	11

$$\int_0^{13} g(x)\,dx \approx 1\cdot g(13) + 5\cdot g(12) + 4\cdot g(7) + 1\cdot g(3) + 2\cdot g(2)$$

$$\approx 1\cdot 11 + 5\cdot 8 + 4\cdot 5 + 1\cdot 8 + 2\cdot 4 \approx 87$$

Midpoint Sums: Find the midpoints of the sub-intervals

$f(x) = x^2 + 1$ on the interval [0, 8] with $n = 4$



$$\int_0^8 f(x)\,dx \approx 2\left[f(1) + f(3) + f(5) + f(7)\right]$$

$$\approx 2\left[2 + 10 + 26 + 50\right]$$

$$\approx 176$$

Midpoint with a chart (3 sub-intervals)

X	0	4	8	12	16	20	24
h(x)	1	5	7	8	10	9	13

$$w = \frac{24-0}{3} = 8$$

$$\int_0^{24} h(x)\,dx \approx 8\left[h(4) + h(12) + h(20)\right]$$

$$\approx 8\left[5 + 8 + 9\right]$$

$$\approx 176$$