4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.

$\int x^n dx$

$$\int \frac{1}{x\sqrt{x}} dx = \int x^{-3/2} dx$$

$$= -2x^{-1/2} + C$$

Differential Equations: finding the general solution or particular solution

$\int f(x)\,dx$

General solution $(+c)$

#7 $\int \frac{dy}{\cancel{dx}} = \int 2(x-1)\,dx$

$y + C_1 = 2\int (x-1)\,dx$

$y + C_1 = 2\left(\frac{x^2}{2} - x\right) + C_2$

$y = x^2 - 2x + C$

Particular solution

#8 $\frac{dy}{dx} = x^2 - 1, (-1, 3)$

$\int dy = \int (x^2 - 1)\,dx$

$y = \frac{x^3}{3} - x + C$

$3 = \frac{-1}{3} + 1 + C$

$\frac{7}{3} = C$

$y = \frac{x^3}{3} - x + \frac{7}{3}$

Solve the differential equation (find the original function)

#9 $f''(x) = \sin x, f'(0) = 1, f(0) = 6$

$f'(x) = \int \sin x \, dx$

$f'(x) = -\cos x + C$

$1 = -\cos 0 + C$

$2 = C$

$f'(x) = -\cos x + 2$

$f(x) = \int (-\cos x + 2) dx$

$f(x) = -\sin x + 2x + C$

$6 = C$

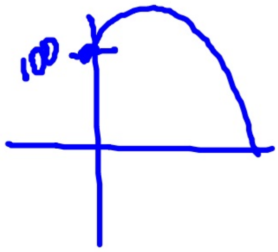
$\boxed{f(x) = -\sin x + 2x + 6}$

$$a(t) = -32 \text{ ft/sec}^2$$

$$v(0) = 36 \text{ ft/sec}$$

$$s(0) = 100 \text{ ft}$$

Solve the differential equation.



$$v(t) = \int -32\,dt$$

$$v(t) = -32t + C$$

$$v(t) = -32t + 36$$

$$s(t) = \int (-32t + 36)\,dt$$

$$s(t) = -16t^2 + 36t + C$$

$$C = 100$$

$$\boxed{s(t) = -16t^2 + 36t + 100}$$

$$s(t) = \frac{1}{2}gt^2 + V_0 t + S_0$$

Motion of a falling object

g : acceleration constant

V_0 : initial velocity

S_0 : initial position (height)

g: -32 ft/sec^2

g: -9.8 m/sec^2

$$s(t) = -16t^2 + V_0 t + S_0$$

$$s(t) = -4.9t^2 + V_0 t + S_0$$

oves along the x-axis in such a way that its acceleration at time t for

$) = \frac{3}{t^3}$. When $t = 1$, the position of the particle is 6 and the velocity is

$v(1) = 2$ $\quad$ $s(1) = 6$

n equation for the velocity, $v(t)$, of the particle for all $t > 0$.

$t) = \int 3t^{-3}\,dt$ $\quad$ $v(t) = \frac{3t^{-2}}{-2} + \frac{7}{2}$

n equation for the position, $x(t)$, of the particle for all $t > 0$.

$) = \int\left(-\frac{3}{2}t^{-2} + \frac{7}{2}\right)dt$ $\quad$ $x(t) = \frac{3}{2}t^{-1} + \frac{7}{2}t + C$

e position of the particle when $t = 2$

$x(t) = \frac{3}{2t} + \frac{7}{2}t + 1$ $\quad$ $6 = \frac{3}{2} + \frac{7}{2} + C$

$1 = C$

$x(2) = \frac{3}{4} + 7 + 1$

$= 8\frac{3}{4}$