4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.

$$y = \int \underbrace{f(x)}_{\text{Integrand}}\, \overbrace{dx}^{\text{Variable of integration}} = \underbrace{F(x)}_{\text{An antiderivative of } f(x)} + \overbrace{C}^{\text{Constant of integration}}.$$

Skip
#51, 53, 55

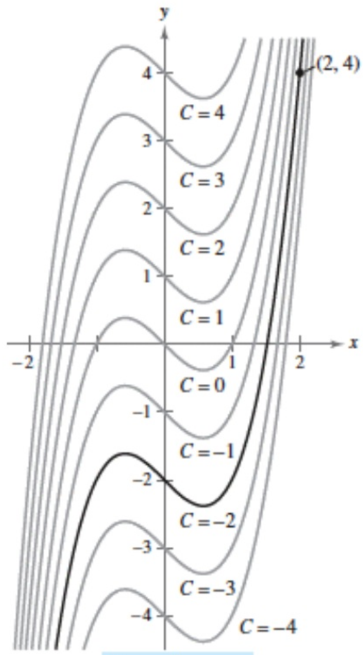
Add
#7

If you know $f'(x)$, how can you find $f(x)$?

$f'(x) = 3x^2-1$
Integrate to
find f(x)

$$\int (3x^2 - 1)\,dx$$

$$x^3 - x + C$$



$F(x) = x^3 - x + C$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$$

$$\int 3x^2\,dx = \cancel{3}\cdot\frac{x^3}{\cancel{3}} + C$$

#1 $\int (8x^3 - 9x^2 + 4)\, dx = \int 8x^3 dx - \int 9x^2 dx + \int 4 dx$

$= \frac{8x^4}{4} - \frac{9x^3}{3} + 4x + C$

$= 2x^4 - 3x^3 + 4x + C$

#2 $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}} \right) dx$

$\int x^{1/2} dx + \int \frac{1}{2} x^{-1/2} dx = \frac{x^{3/2}}{3/2} + \frac{1}{2} \cdot \frac{x^{1/2}}{1/2} + C$

$= \frac{2}{3} x^{3/2} + x^{1/2} + C$

#3 $\int \frac{x^2+2x-3}{x^4}\,dx = \int x^{-4}(x^2+2x-3)\,dx$

$$= \int (x^{-2}+2x^{-3}-3x^{-4})\,dx$$

$$\frac{x^{-1}}{-1}+\frac{2x^{-2}}{-2}-\frac{3x^{-3}}{-3}+C$$

$$-\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}+C$$

Integration Rules

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

Power Rule

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

#4 $\int \sec y(\tan y - \sec y)\, dy$

$$\int (\sec y \tan y - \sec^2 y)\, dy = \sec y - \tan y + C$$

#5 $\int \frac{\sin x}{(1 - \sin^2 x)} dx$

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx$$

$$\int \tan x \sec x\, dx = \sec x + C$$

$$\int (1+\cot^2 x)\,dx$$

$$\int \csc^2 x\,dx = -\cot x + C$$

Differential Equations: finding the general solution or particular solution (find the original function)

General solution

#6 $\frac{dy}{dx} = 2(x-1)$

$y = 2\left(\frac{1}{2}x^2 - x\right) + C$

or

$\boxed{y = x^2 - 2x + C}$

Particular solution

#7 $\frac{dy}{dx} = x^2 - 1, (-1, 3)$

$y = \frac{1}{3}x^3 - x + C$

$3 = -\frac{1}{3} + 1 + C$

$\frac{7}{3} = C$

$y = \frac{1}{3}x^3 - x + \frac{7}{3}$

Solve the differential equation (find the original function)

#8 $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$

$f'(x) = -\cos x + C$

$1 = -\cos 0 + C$

$2 = C$

$f'(x) = -\cos x + 2$

$f(x) = -\sin x + 2x + C$

$6 = -\sin 0 + 0 + C$

$6 = C$

$\boxed{f(x) = -\sin x + 2x + 6}$

oves along the x-axis in such a way that its acceleration at time t for

$) = \frac{3}{t^3}$. When $t = 1$, the position of the particle is 6 and the velocity is

$s(1) = 6 \quad v(1) = 2$

an equation for the velocity, $v(t)$, of the particle for all $t > 0$.

an equation for the position, $x(t)$, of the particle for all $t > 0$.

e position of the particle when $t = 2$

$v(t) = -\frac{3}{2}t^{-2} + \frac{7}{2}$

$s(t) = \frac{3}{2}t^{-1} + \frac{7}{2}t + 1$