Local Linear Approximation
aka
Tangent Line Approximation

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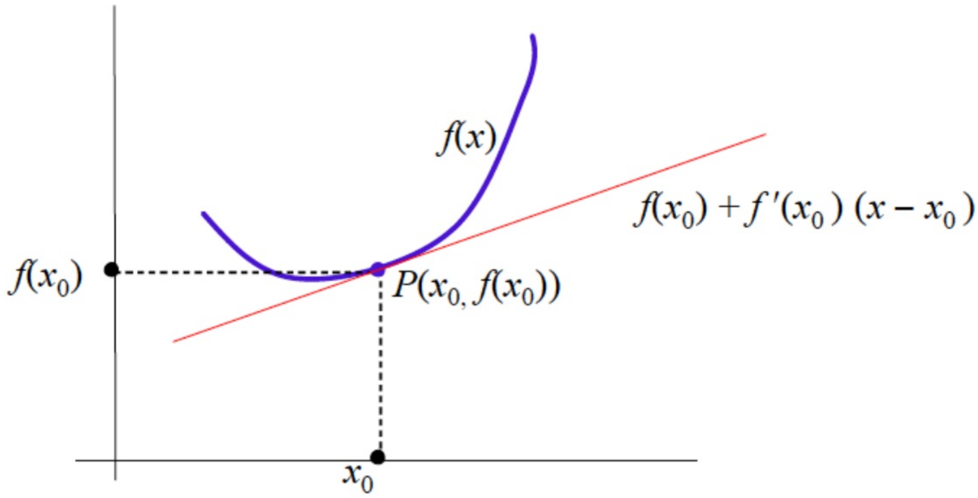
The equation of the tangent line to the graph of the function $f(x)$ at the point x_0 is $y - f(x_0) = f'(x_0)(x - x_0)$



7.762392

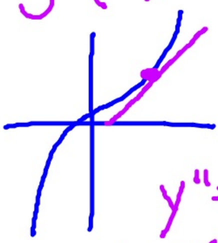
Example.Use local linear approximations to approximate the quantity$(1.98)^3$ ≈ 7.76

Is this an over or under approximation?

$y = x^3$

$y' = 3x^2$

$y'(2) = 12$



$y''(2) > 0$

under approx.

$y'' = 6x$

$y''(2) > 0$

$(\underset{x}{1.98}, \underset{y}{\_\_\_\_})$ $(\underset{x_1}{2}, \underset{y_1}{8})$

$y - y_1 = m(x - x_1)$

$y = m(x - x_1) + y_1$

$y \approx 12(1.98 - 2) + 8$

$y \approx 12(-.02) + 8$

$\approx 12\left(\frac{-2}{100}\right) + 8 = \frac{-24}{100} + 8 \approx 7\frac{76}{100}$

7.76

Example.Use local linear approximations to approximate the quantity $\sqrt{80.9}$

Is this an over or under approximation?

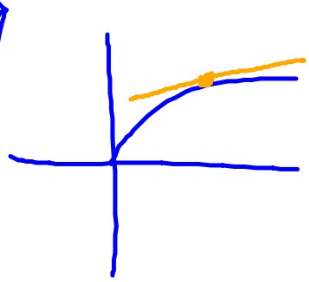
$y = \sqrt{x}$ $\qquad$ $(80.9, \_\_\_)$ $(81, 9)$

$y' = \frac{1}{2\sqrt{x}}$ $\qquad$ $y \approx \frac{1}{18}(80.9 - 81) + 9$

$y'(81) = \frac{1}{18}$ $\qquad$ $y \approx \frac{1}{18}\left(-\frac{1}{10}\right) + 9$

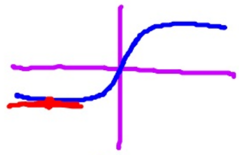
$y''(81) < 0$ $\qquad$ $y \approx \frac{-1}{180} + 9$

over-approx. $\qquad$ $y \approx 8\frac{179}{180}$



Approximate the cube root of -65.

Is this an over or under approximation?



$y = \sqrt[3]{x}$

$y' = \frac{1}{3}x^{-2/3}$

$y'(-64) = \frac{1}{3}(-64)^{-2/3}$

$= \frac{1}{3} \cdot 64^{-2/3}$

$= \frac{1}{3}(4^3)^{-2/3}$

$= \frac{1}{48}$

$(\underset{x}{-65}, \underset{y}{\_\_\_})$ $(\underset{x_1}{-64}, \underset{y_1}{-4})$

$y \approx \frac{1}{48}(-65 - -64) - 4$

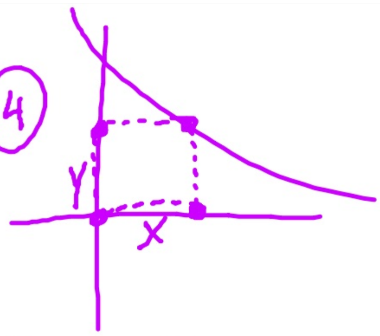
$y \approx \frac{1}{48}(-1) - 4 \approx -4\frac{1}{48}$

$y'' = \frac{-2}{9}x^{-5/3}$

$\rightarrow y''(-64) > 0$ ccu

under approx

4



Primary

$$A = xy$$

$$A = x\left(\frac{4-x}{x+2}\right)$$

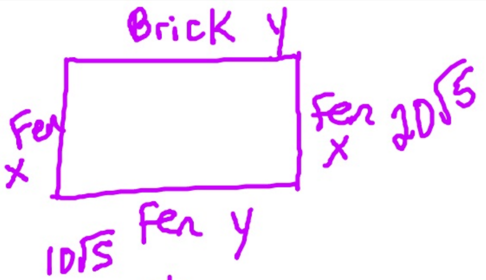
Secondary

$$y = \frac{4-x}{x+2}$$

$$A = \frac{4x-x^2}{x+2}$$

1 $x + 4\left(\frac{1}{x}\right)^2 = \text{min}$

(2)



$A = 1000$

$xy = 1000$

$y = \frac{1000}{x}$

min. cost

$$C = 2x \cdot 10 + y \cdot 10 + y \cdot 30$$

$$C = 20x + 40y$$

$$C = 20x + 40\left(\frac{1000}{x}\right) = 20x + \frac{40000}{x}$$

$$C' = 20 - \frac{40000}{x^2} = \frac{20x^2 - 40000}{x^2} \; ; \quad 20x^2 = 40000$$

$$x^2 = 2000$$

$$x = \sqrt{2000}$$

$$x = 20\sqrt{5}$$

$$C'' = 80000x^{-3} \qquad C''(\sqrt{2000}) > 0$$