More Optimization

Because d (distance) is the smallest when the expression inside the radical is smallest, you need only find the critical numbers of the radicand.

Which point(s) on y = $x^2 + 1$ are closest to (0,2)?

minimize distance

secondary

Primary

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (y-2)^2}$$

$$d = \sqrt{x^2 + (x^2+1-2)^2}$$

$$d = \sqrt{x^2 + (x^2-1)^2}$$

$$D = x^2 + (x^2-1)^2$$

$$D' = 2x + 2(x^2-1)\cdot 2x$$

$$D' = 2x + 4x^3 - 4x$$

$$D' = 4x^3 - 2x$$

$$0 = 2x(2x^2-1)$$

$$\cancel{x=0}, \pm\sqrt{\tfrac{1}{2}}$$

$- \quad + \quad - \quad +$

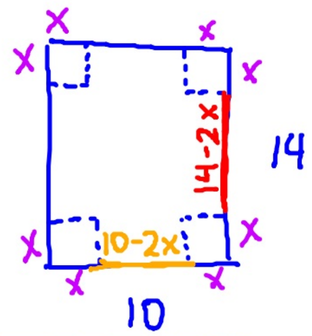
$-\sqrt{\tfrac{1}{2}} \quad 0 \quad \sqrt{\tfrac{1}{2}}$

$$\left(\sqrt{\tfrac{1}{2}}, \tfrac{3}{2}\right)$$

$$\left(-\sqrt{\tfrac{1}{2}}, \tfrac{3}{2}\right)$$

Find the maximum volume of a box that can be made by cutting out squares from the corners of a 14 inch by 10 inch rectangular sheet of cardboard and folding up the sides.

$$V = (10-2x)(14-2x)x \qquad \boxed{120.1 \text{ in}^3}$$



$$V = 2(5-x)2(7-x)x$$

$$V = 4(5-x)(7x-x^2)$$

$$V = 4(x^3-5x^2-7x^2+35x)$$

$$V = 4(x^3-12x^2+35x)$$

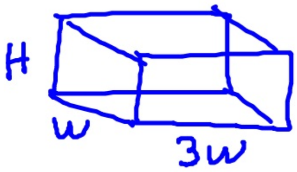
$$V' = 4(3x^2-24x+35)$$

$$x = \frac{24 \pm \sqrt{24^2-4(3)(35)}}{6}$$

$$x = \frac{24 \pm \sqrt{156}}{6} \approx \cancel{6.082}, \ 1.918$$

We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10 per square foot and the material used to build the sides cost $6 per square foot. If the box must have a volume of 50 cubic feet, determine the width that will minimize the cost to build the box.

$L = 3W$



Primary

$$C = 2(3W^2)\cdot \overset{\$}{10} + 2HW\overset{\$}{(6)} + 2(3WH)\overset{\$}{6}$$

$$C = 60W^2 + 12HW + 36WH$$

$$C = 60W^2 + \frac{200}{W} + \frac{600}{W}$$

$$C = 60W^2 + \frac{800}{W}$$

$$C' = \frac{120W}{1} - \frac{800}{W^2}$$

$$\frac{120W^3 - 800}{W^2} \; ; \; \sqrt[3]{\frac{20}{3}}$$

Secondary

$$V = LWH$$

$$V = 3WWH$$

$$V = 3W^2H$$

$$50 = 3W^2H$$

$$\frac{50}{3W^2} = H$$

Find the dimensions of a rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

$$A = 2xy$$

$$A = 2x(8 - x^2)$$

$$A(x) = 16x - 2x^3$$

$$A'(x) = 16 - 6x^2$$

$$0 = 2(8 - 3x^2)$$

$$x = \pm\sqrt{\frac{8}{3}}$$

$$y = 8 - \left(\sqrt{\frac{8}{3}}\right)^2$$

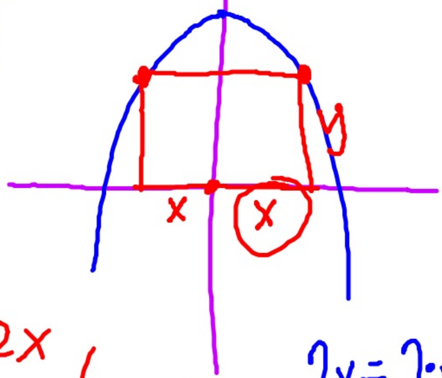
$$y = 8 - \frac{8}{3}$$

$$y = \frac{16}{3}$$

$$A''(x) = -12x$$

$$f''\left(\sqrt{\frac{8}{3}}\right) < 0 \checkmark \text{ max}$$

$$f''\left(-\sqrt{\frac{8}{3}}\right) > 0$$



$$2x = 2\cdot\sqrt{\frac{8}{3}}$$

$$\frac{2\sqrt{8}}{\sqrt{3}}$$

$$\boxed{\frac{2\sqrt{8}}{\sqrt{3}} \times \frac{16}{3}}$$

A box with a square base and open top must have a volume of 32,000 cubic centimeters. Find the dimensions of the box that minimize the amount of material used.