3.7 Optimization Problems

- Solve applied minimum and maximum problems.

<u>Steps to follow</u>

1. Identify what you are trying to maximize or minimize.
2. Determine the primary and secondary equation. The primary will be the one you are trying to optimize.
3. Take the derivative of the primary equation. Verify there is an absolute max or min.
4. Reread the question to make sure you answered the question

#1 Find two positive numbers that satisfy the given requirements.

The sum of the first number squared and the second number is 54 and the product is a maximum.

Max product (primary) | secondary

$P = xy$

$x^2 + y = 54$

$y = 54 - x^2$

$P = x(54 - x^2)$

$P(x) = 54x - x^3$

$P'(x) = 54 - 3x^2$

$0 = 54 - 3x^2$

$\pm\sqrt{18} = x$

Since P(x) is increasing on (0, sqrt18) and decreasing on (sqrt18, inf), sqrt 18 is an absolute max

$\boxed{\sqrt{18},\ 36}$

#2: Find the length and width of a rectangle that has a perimeter of 50 cm maximizes the area.

max area

primary

$A = LW$

$A(L) = L(25 - L)$

$A(L) = 25L - L^2$

$A'(L) = 25 - 2L$

$\frac{25}{2} = L$

E —+—|—−—→ A'

12.5

secondary

$2L + 2W = 50$

$L + W = 25$

$W = 25 - L$

$$\boxed{L = \frac{25}{2}\text{ cm} \quad W = \frac{25}{2}\text{ cm}}$$

#4: A rectangular page is to contain 24 square inches of print. The margins on the top and bottom are to be 1.5 inches and the margins on the left and right are to be 1 inch. what should be the dimensions of the page so that the least amount of paper is used.

minimize Area

$A = LW$

$A = (x+2)(y+3)$

$A(x) = (x+2)\left(\frac{24}{x}+3\right)$

$A(x) = 24 + 3x + \frac{48}{x} + 6$

$A'(x) = 3 - 48x^{-2}$

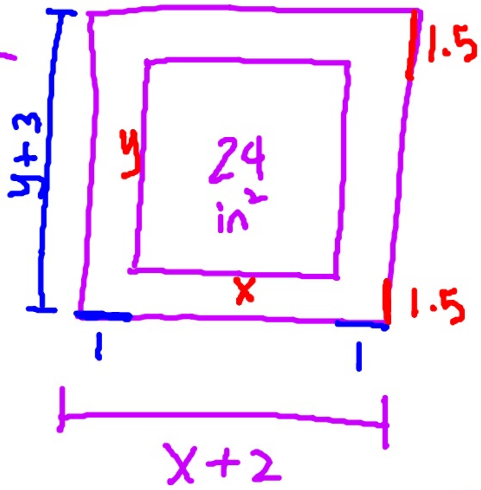
$= 3 - \frac{48}{x^2} = \frac{3x^2-48}{x^2} = 0;\ x = 4$

secondary

$A = xy$

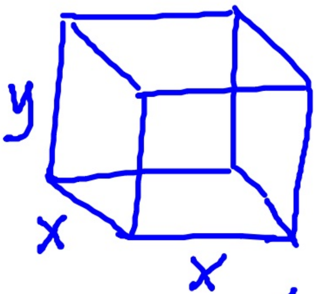
$24 = xy$

$\frac{24}{x} = y$



$\boxed{6\text{in} \times 9\text{in}}$

#3 A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a maximum volume?



Primary: $V = x^2 y$

Secondary: $A = x^2 + 4xy$

$108 = x^2 + 4xy$

$\dfrac{108 - x^2}{4x} = y$

$V(x) = x^2\left(\dfrac{108-x^2}{4x}\right)$

$V(x) = \frac{1}{4}x\left(108 - x^2\right)$

$V(x) = \frac{1}{4}\left(108x - x^3\right)$

$V'(x) = \frac{1}{4}\left(108 - 3x^2\right)$; $x = 6$

$V''(x) = \frac{1}{4}(-6x)$

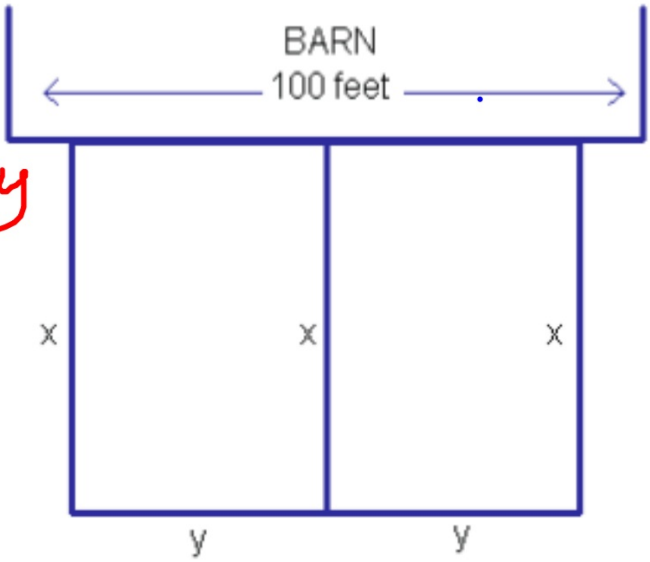
$V''(6) = \frac{1}{4}(-36) < 0_{max}$

6in x 6in x 3in

#5: Two pens are to be built alongside a barn as shown.The barn will make up one side of each pen. If 200 ft of fencing are available,what pen size maximizes the area?

Primary: $A = 2xy$

Secondary: $200 = 3x + 2y$



$x = \frac{100}{3}$ ft

$y = 50$ ft

100/3 ft by 50 ft