3.5 Limits at Infinity

- Determine (finite) limits at infinity.
- Determine the horizontal asymptotes, if any, of the graph of a function.
- Determine infinite limits at infinity.

Limits at Infinity

This section discusses the "end behavior" of a function on an *infinite* interval.

$x \to \infty$
$x \to -\infty$

Horizontal Asymptotes

In Figure 3.34, the graph of f approaches the line $y = L$ as x increases without bound. The line $y = L$ is called a **horizontal asymptote** of the graph of f.

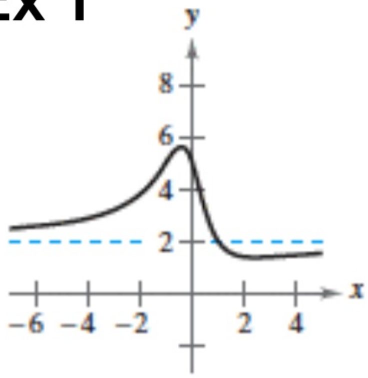
DEFINITION OF A HORIZONTAL ASYMPTOTE

The line $y = L$ is a **horizontal asymptote** of the graph of f if

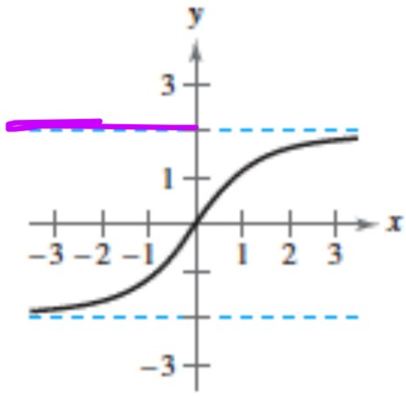
$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L.$$

Find the limit as x approaches infinity and negative infinity

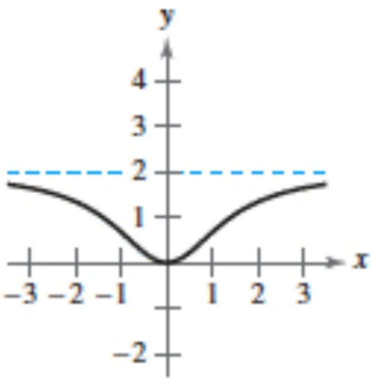
Ex 1



Ex 2



Ex 3



$\lim_{x \to \infty} f(x) = 2$

$\lim_{x \to -\infty} f(x) = 2$

GUIDELINES FOR FINDING LIMITS AT $\pm\infty$ OF RATIONAL FUNCTIONS

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

Ex 4

(a) $\lim\limits_{x\to\infty} \dfrac{3-2x}{3x^3-1} = 0$ Bobo

(b) $\lim\limits_{x\to\infty} \dfrac{3-2x}{3x-1} = \dfrac{-2}{3}$ Eats d.c.

(c) $\lim\limits_{x\to\infty} \dfrac{3-2x^2}{3x-1}$ $-\infty$ Botn

1000 1000

∞ or $-\infty$

(c) $\lim\limits_{x \to \infty} \dfrac{3-2x^2}{3x-1}$

$$\lim_{x \to \infty} \frac{\frac{3}{x} - \frac{2x^2}{x}}{\frac{3x}{x} - \frac{1}{x}}$$

$$\lim_{x \to \infty} \frac{\overset{\to 0}{\frac{3}{x}} - 2x}{3 - \underset{\to 0}{\frac{1}{x}}} = \lim_{x \to \infty} \frac{-2x^{1000}}{3}$$

$-\infty$

Find the limit.

Ex 5

$$\lim_{x\to-\infty}\left(\frac{5}{x}-\frac{x}{3}\right) = \lim_{x\to-\infty}\frac{15-x^2}{3x} \quad \frac{-}{-} \quad \infty$$

x^2: -1000; $3x$: -1000

Ex 6

$$\lim_{x\to-\infty}\left(\frac{1}{2}x-\frac{4}{x^2}\right) = \lim_{x\to-\infty}\frac{x}{2} - \lim_{x\to-\infty}\frac{4}{x^2}$$

$$-\infty - 0$$

$$-\infty$$

Ex 7

nonexistent

$$\lim_{x \to \infty} \sin x$$

does not exist

Ex 8*

$$\lim_{x \to \infty} \frac{\sin x}{x_{1000}} = 0$$

$\frac{1, -1}{1000}$

$\frac{1}{1000}$ or $\frac{-1}{1000}$

$$\lim_{x \to \infty} \sin x \cdot \frac{1}{x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Ex 9

$$\lim_{x \to \infty} \cos \frac{1}{x_{1000}} = 1$$

$\cos 0$

Ex 10

Find the two horizontal asymptotes for :

$$f(x) = \frac{20x}{\sqrt{9x^2 - 1}}$$

Find the limit as x approaches infinity and negative infinity.

$$\lim_{x \to \infty} \frac{20x}{\sqrt{9x^2 - 1}}$$

$$\frac{20}{\sqrt{9}}$$

$$\frac{20}{3}$$

$$\lim_{x \to -\infty} \frac{20x}{\sqrt{9x^2 - 1}}$$

$$-\frac{20}{3}$$

$$\lim_{x \to \infty} \frac{x}{\sqrt{5x^2+1}}$$

$$\frac{1}{\sqrt{5}}$$

$$\lim_{x \to -\infty} \frac{4x}{\sqrt{9x^2+13}}$$

$$-\frac{4}{3}$$

$$\lim_{x \to -\infty} \frac{-x}{\sqrt{4x^2+7}}$$

$$\frac{1}{2}$$