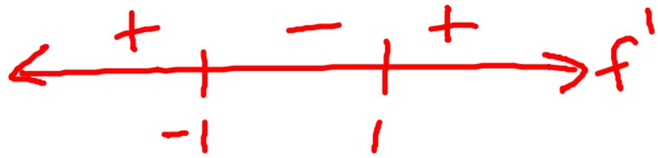
There are two tests that have an official name

First Derivative Test



Second Derivative Test

BOTH tests are ways to locate relative extrema

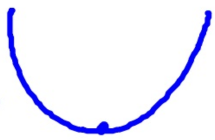
local

THEOREM 3.9 SECOND DERIVATIVE TEST

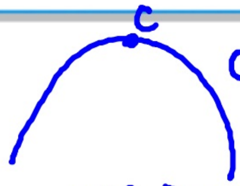
Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then f has a relative minimum at $(c, f(c))$.
2. If $f''(c) < 0$, then f has a relative maximum at $(c, f(c))$.

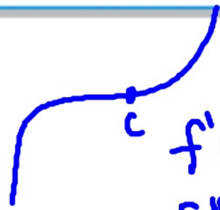
If $f''(c) = 0$, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test.



$f'(c) = 0$
$f''(c) > 0$
$f(c)$: rel. min.



$f'(c) = 0$
$f''(c) < 0$
$f(c)$: rel max



$f'(c) = 0$
$f''(c) = 0$
test fails

Find the relative extrema using the second derivative test.

#1 $f(x) = -3x^5 + 5x^3$

$f'(x) = -15x^4 + 15x^2$

$0 = -15x^2(x^2 - 1)$

$x = 0, 1, -1$

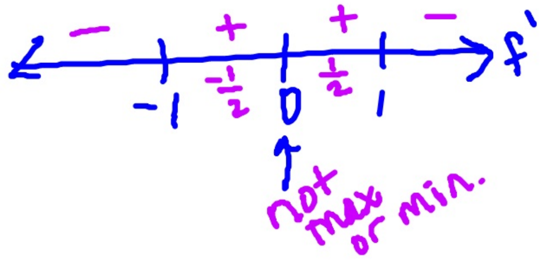
$f''(x) = -60x^3 + 30x$

$f''(x) = -30x(2x^2 - 1)$

$f''(1) < 0$; rel. max $(1, 2)$

$f''(-1) > 0$; rel. min $(-1, -2)$

$f''(0) = 0$; test fails



$f'(6) = 0$ $\quad \cap \quad \cup$

$f''(6) = 4 \xrightarrow[f'' > 0]{\text{ccu}}$

$f(6)$: rel. min.

relative extrema (1st der test or 2nd deriv)

intervals of increasing/decreasing

intervals of concavity

points of inflection

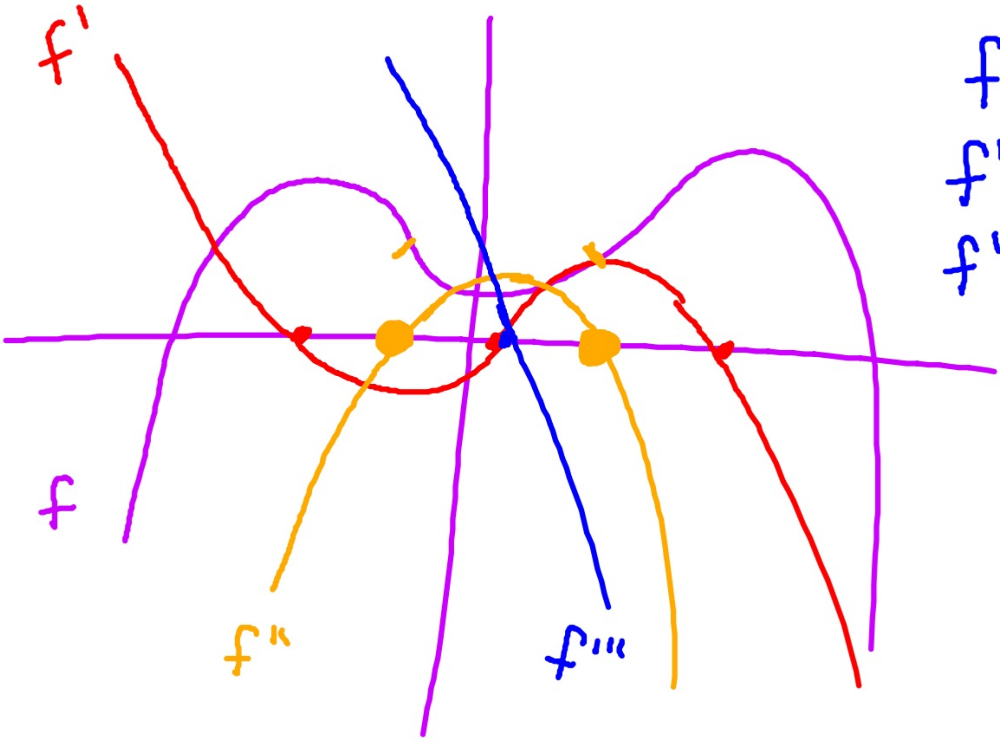
sketching f' and f'' graphs

Justifying!!!

Finding rel. extrema : $f(x) = \sin 4x$

Finding and simplifying : $f(x) = x\sqrt{x+3}$

$f(x) = x(3x-1)^4$



f' : velocity

f'' : accel.

f''' : jerk

(11) $f(x) = 24(x^2+12)^{-1}$

$f'(x) = -24(x^2+12)^{-2} \cdot 2x = \underline{-48x(x^2+12)^{-2}}$

$f''(x) = -48\left(x \cdot -2(x^2+12)^{-3} \cdot 2x + (x^2+12)^{-2} \cdot 1\right)$

$= +48\left(+(x^2+12)^{-3}\left(4x^2 - (x^2+12)^1\right)\right)$

$f''(x) = \dfrac{48(3x^2-12)}{(x^2+12)^3}$

f'': $+$ at -10 | -2 | $-$ at 0 | 2 | $+$ at 10

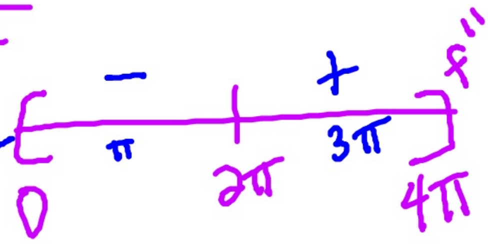
POI: $x = 2, -2$

31) $f(x) = \sin\frac{x}{2}$ $\quad [0, 4\pi]$

$f'(x) = \cos\frac{1}{2}x \cdot \frac{1}{2}$

$f''(x) = -\frac{1}{2}\sin\left(\frac{1}{2}x\right)\cdot\frac{1}{2}$

$f''(x) = -\frac{1}{4}\sin\left(\frac{1}{2}x\right)$



$0 = \sin\left(\frac{1}{2}x\right)$

$\frac{1}{2}x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = \cancel{0}, 2\pi, \cancel{4\pi}, \cancel{6\pi}$

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$$f(x) = x(x-4)^3$$

$$f'(x) = x \cdot 3(x-4)^2 + (x-4)^3$$

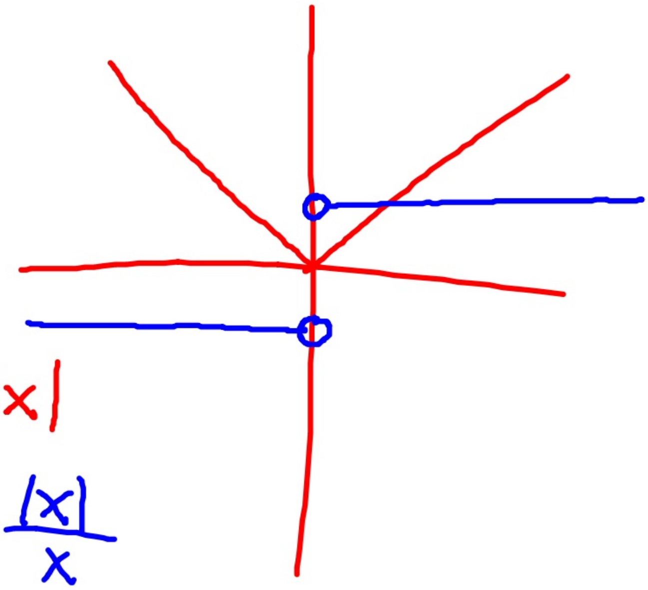
$$= (x-4)^2(3x + x - 4) = (x-4)^2(4x-4)$$

$$= 4(x-4)^2(x-1)$$

$$f''(x) = 4\left[(x-4)^2 \cdot 1 + (x-1) \cdot 2(x-4)^1\right]$$

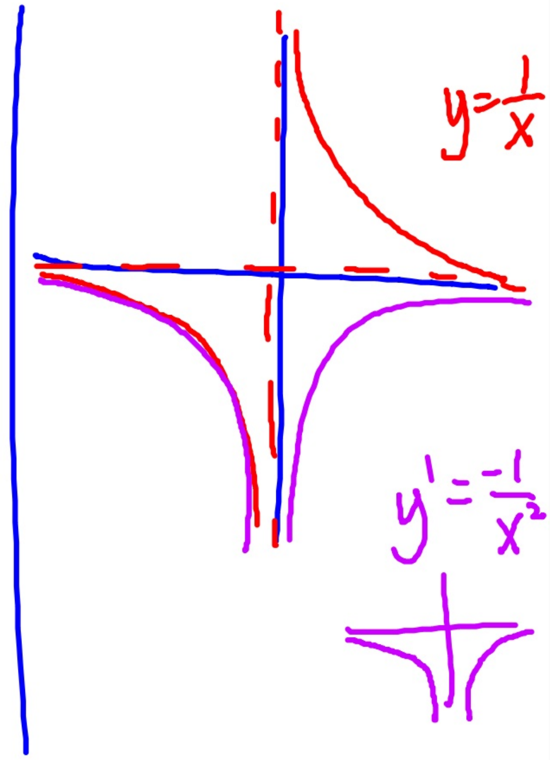
$$= 4\left[(x-4)\left[(x-4) + 2(x-1)\right]\right] = 4(x-4)(3x-6)$$

f'': $+$ (0) | 2 | $-$ (3) | 4 | $+$



$y = |x|$

$y' = \frac{|x|}{x}$



$y = \frac{1}{x}$

$y' = \frac{-1}{x^2}$