3.3 Increasing and Decreasing Functions and the First Derivative Test

- Determine intervals on which a function is increasing or decreasing.
- Apply the First Derivative Test to find relative extrema of a function.

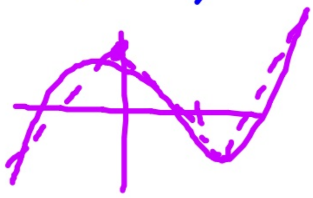
THEOREM 3.5 TEST FOR INCREASING AND DECREASING FUNCTIONS

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

Find the intervals where the function is increasing or decreasing. Justify your answer.

$D: (-\infty, \infty)$



#1 $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$x = 0, 4$

$+ \quad - \quad +$

f'

$0 \quad 4$

incr. decr. incr.

f

$0 \quad 4$

f is increasing on (-inf, 0) U (4, inf) because f'>0 on these intervals

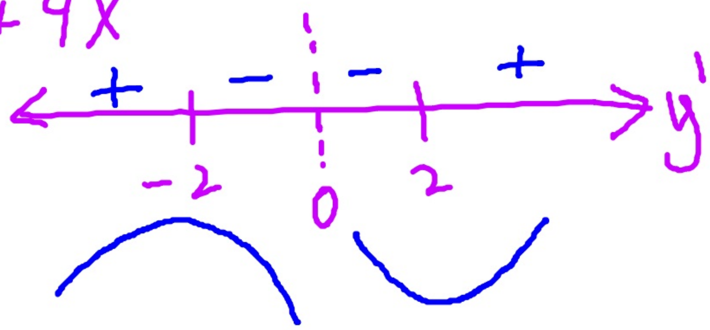
f is decreasing (0, 4) because f' < 0 on this interval

Find the intervals where the function is increasing or decreasing.

$D: (-\infty, 0) \cup (0, \infty)$

#2

$$y = x + \frac{4}{x} = x + 4x^{-1}$$



$$y' = 1 - 4x^{-2}$$

$$= 1 - \frac{4}{x^2}$$

$$y' = \frac{x^2 - 4}{x^2} \; ; \; x = \pm 2$$

y is increasing on (-inf,-2) U(2,inf)
because y'>0 on these intervals

y is decreasing on (-2, 0) U (0,2)
because y'<0 on this interval