3.2 Rolle's Theorem and the Mean Value Theorem

- Understand and use Rolle's Theorem.
- Understand and use the Mean Value Theorem.

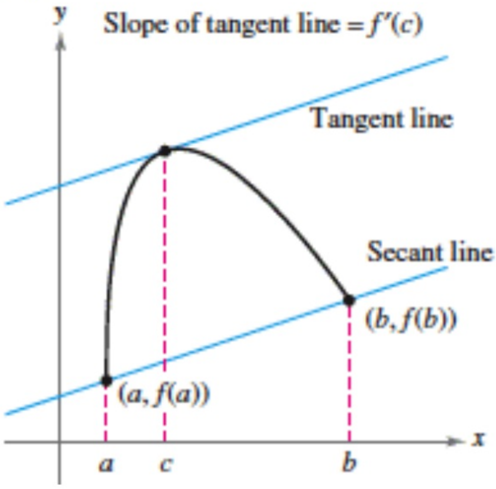
THEOREM 3.4 THE MEAN VALUE THEOREM MVT

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

cont. $[a,b]$ diff (a,b)

$m_{tan} = m_{sec}$



Verify if MVT Theorem is applicable. If so, find the value(s) c that satisfy the MVT.

#1 $f(x) = x^4 - 8x, \quad [0, 2]$

Cont. $[0,2]$ ✓

diff $(0,2)$ ✓

$f'(x) = 4x^3 - 8$

$f'(c) = 4c^3 - 8$

$\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{0}{2}$

$= 0$

$m_{tan} = m_{sec}$

$4c^3 - 8 = 0$

$c^3 = 2$

$\boxed{c = \sqrt[3]{2}}$

#2 $f(x) = \frac{x+1}{x}$, $[-1, 2]$

Verify

cont. $[-1, 2]$ No.

MVT does not apply

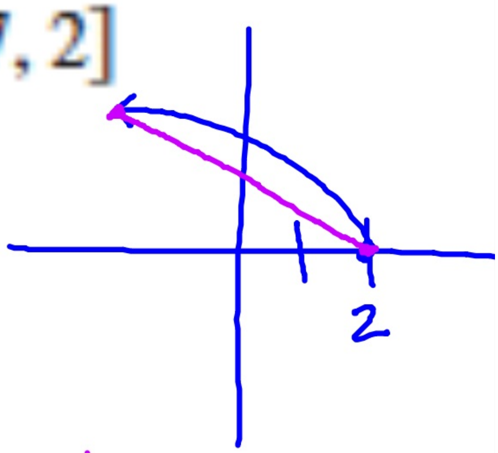
#3 $f(x) = \sqrt{2 - x}, \quad [-7, 2]$

Cont. $[-7, 2]$ ✓

diff $(-7, 2)$ ✓

$m_{sec} = -\frac{1}{3}$

$m_{tan} = -\frac{1}{2\sqrt{2-x}}$



$+\frac{1}{3} = \frac{+1}{2\sqrt{2-c}}$

$3 = 2\sqrt{2-c}$

$9 = 4(2-c)$

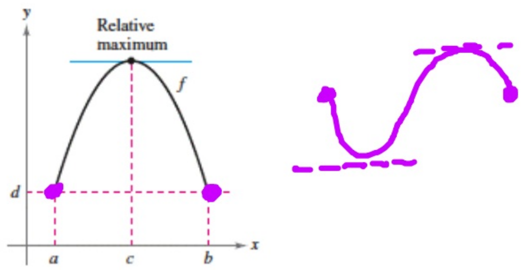
$c = -1/4$

THEOREM 3.3 ROLLE'S THEOREM

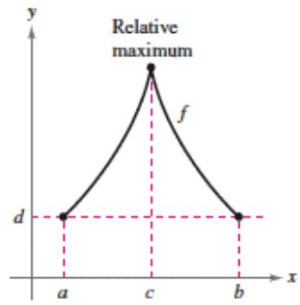
Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). If

$$f(a) = f(b)$$

then there is at least one number c in (a, b) such that $f'(c) = 0$.



(a) f is continuous on $[a, b]$ and differentiable on (a, b).



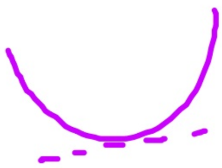
(b) f is continuous on $[a, b]$.

Verify if Rolle's Theorem applies.
Then find the value(s) where
$f'(c) = 0$

#4 $f(x) = x^2 - 5x + 4, \quad [1, 4]$

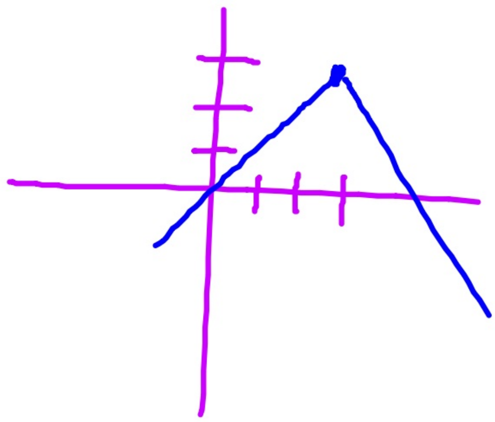
Cont. $[1,4]$ ✓
diff $(1,4)$
$f(1) = f(4)$?
$0 = 0$ ✓

$m_{tan} = 0$
$2x - 5 = 0$
$\boxed{x = 5/2}$



Verify if Rolle's Theorem applies.
Then find the value(s) where
f'(c) = 0

#5 $f(x) = 3 - |x - 3|, \quad [0, 6]$



cont [0,6] ✓

diff (0,6) X

Rolle's does not apply.

#6 $f(x) = \dfrac{x^2 - 1}{x}, \quad [-1, 1]$

Cont. $[-1, 1]$ X

disc. @ $x = 0$

Rolle's does not apply.

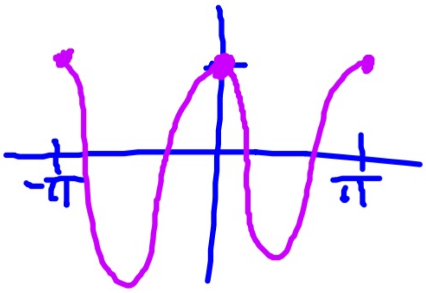
#7 $f(x) = \cos 2x, \quad [-\pi, \pi]$

Cont. $[-\pi, \pi]$ ✓

diff $(-\pi, \pi)$ ✓

$f(-\pi) = f(\pi)$

$1 = 1$ ✓



$m_{tan} = 0$

$-2\sin 2x = 0$

$\sin 2x = 0$

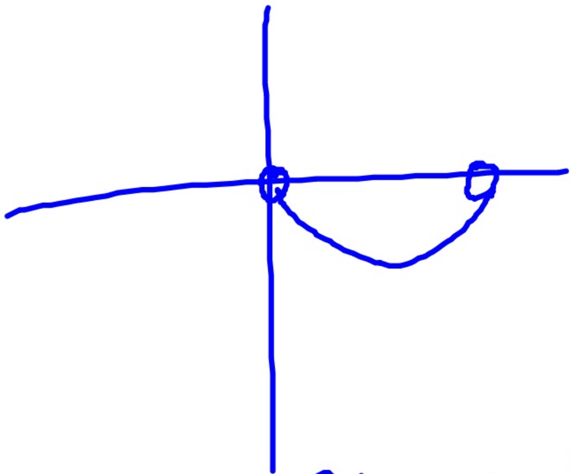
$A = 2x \qquad \sin A = 0$

$A = -\pi, 0, \pi, 2\pi, 3\pi, 4\pi.$

$2x = -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

$x = -\frac{\pi}{2}, 0, \frac{\pi}{2}, \cancel{\pi}, \cancel{\frac{3\pi}{2}}, \cancel{2\pi}$

39.) $f(x) = x^2 - 2x$ c.) $(0, 2)$



Abs. min $(1, -1)$

No Abs max.

33.) $f(x) = \cos(\pi x)$ $[0, 1/6]$

$f'(x) = -\pi \sin(\pi x)$

$0 = -\pi \sin(\pi x)$

$0 = \sin(\pi x)$ $A = \pi x$

$0 = \sin A$

$A = 0, \pi, 2\pi, 3\pi, 4\pi$

$\pi x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, 1, 2, 3, 4$

	x	$f(x)$
max	0	1
min	$1/6$	$\sqrt{3}/2$