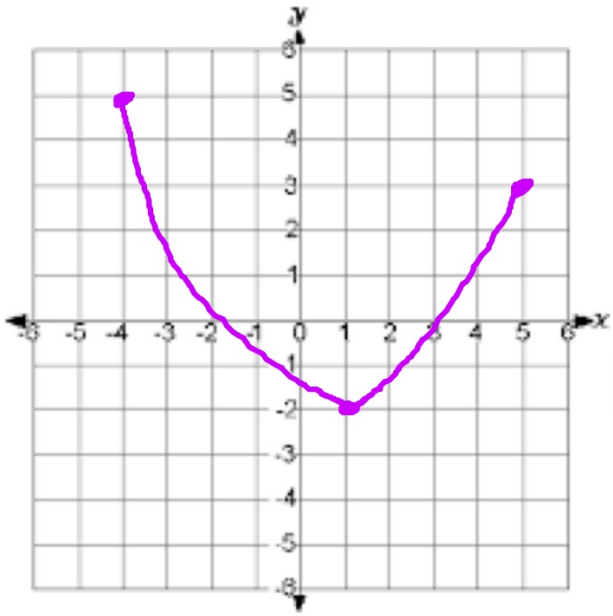
3.1 Extrema on an Interval

- Understand the definition of extrema of a function on an interval.
- Understand the definition of relative extrema of a function on an open interval.
- Find extrema on a closed interval.

Is there an absolute min? max?



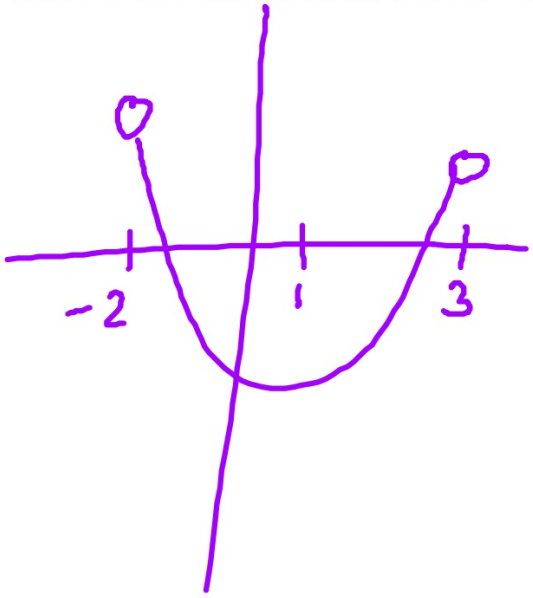
Abs min $x = 1$

Abs. min value $y = -2$

Abs. max $x = -4$

Abs. max value: $y = 5$

Is there an absolute min? max?

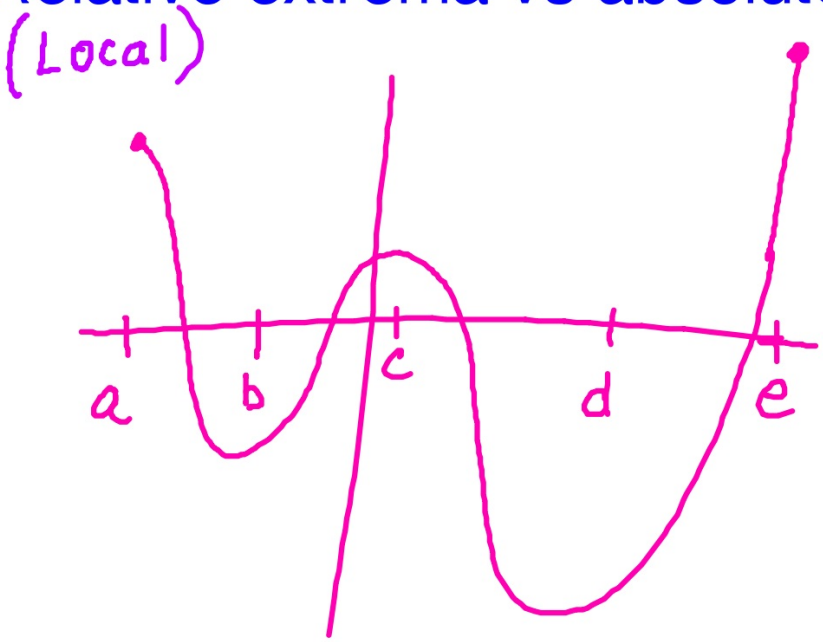


$(-2, 3)$

Yes; abs. min

No, abs. max

Relative extrema vs absolute extrema [a, e]



rel. min: $x = b, d$

rel. max; $x = c$

abs. min: $x = d$

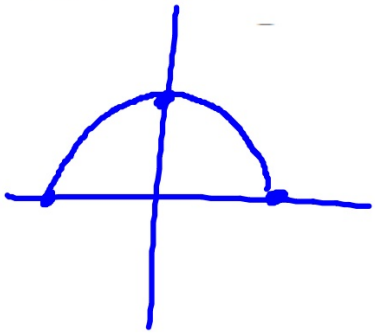
abs. max: $x = e$

Relative extrema is NEVER at the endpoints

Locate the absolute extrema (if any) on the stated intervals

$f(x) = \sqrt{4 - x^2}$

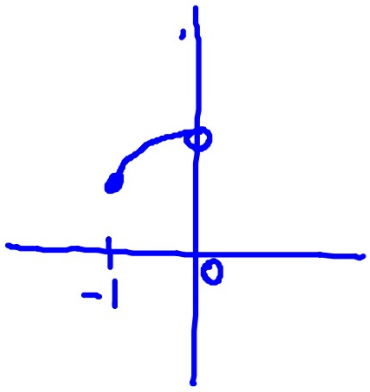
a) [-2, 2]



Abs. max value : 2

Abs. min Value : 0

b. [-1, 0)

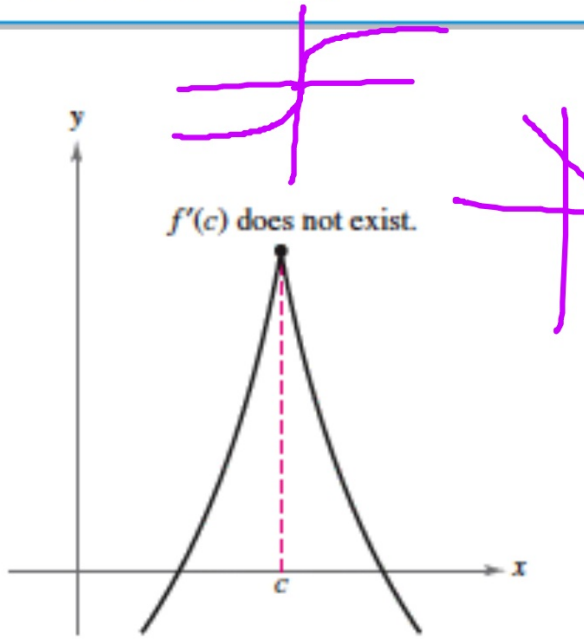


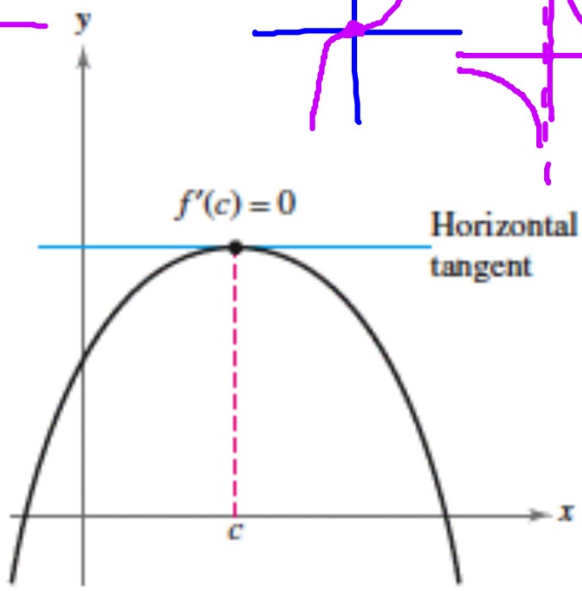
Abs. max value : none

Abs. min value : $\sqrt{3}$

DEFINITION OF A CRITICAL NUMBER

Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f.





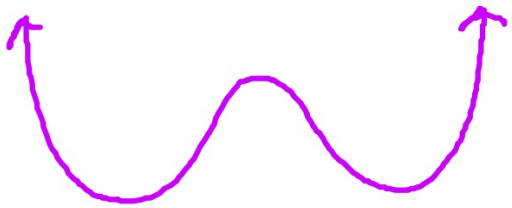
#1 Find the critical numbers

$g(x) = x^4 - 4x^2$ D: $(-\infty, \infty)$

$g'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$

$x = 0, \pm\sqrt{2}$

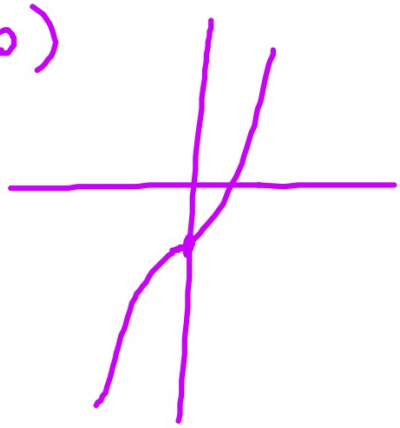


#2 $f(x) = 2x^3 - 1$ $(-\infty, \infty)$

$f'(x) = 6x^2$

$0 = 6x^2$

$0 = x$



#3 $f(x) = 2x - 3x^{2/3}$

D: $(-\infty, \infty)$

$f'(x) = 2 - 2x^{-1/3}$

Make the derivative useful

$= \frac{2}{1} - \frac{2}{x^{1/3}}$

$= \frac{2x^{1/3} - 2}{x^{1/3}}$

top = 0 (zero slope)

$2x^{1/3} = 2$

$x = 1$

bottom = 0 (undefined slope)

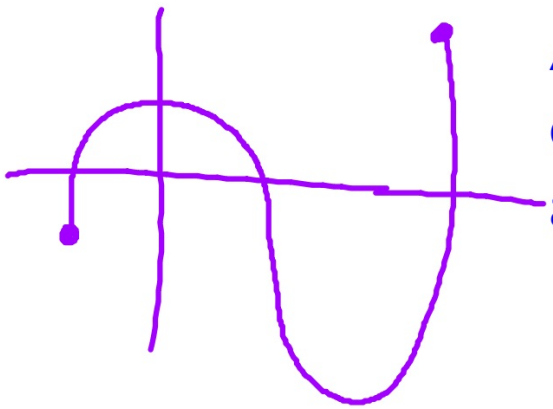
$x^{1/3} = 0$

$x = 0$

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these values is the minimum. The greatest is the maximum.



Absolute extrema:
check endpoints
and critical points

#4: Find the absolute extrema.

points

$$f(x) = \frac{2x}{x^2 + 1}, \ [-2, 2]$$

Check endpoints and critical points

$$f'(x) = \frac{-2x^2 + 2}{(x^2 + 1)^2}$$

X	y
-2	-4/5
-1	-1
1	1
2	4/5

Abs. max @ (1, 1)

Abs. min @ (-1, -1)

#5: Find the absolute extrema.

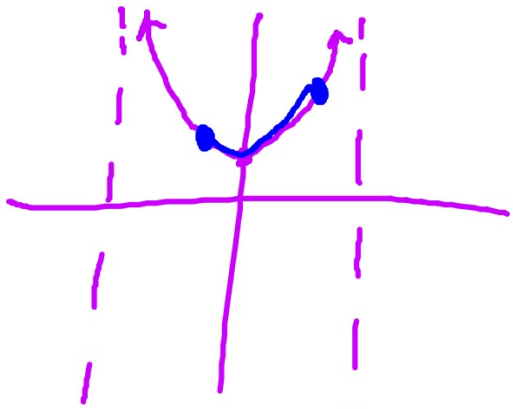
$$g(x) = \sec x, \left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$$

$g'(x) = \sec x \tan x$

$0 = \sec x \tan x$

~~$\sec x = 0$~~ $\tan x = 0$

$x = 0$



x	y	
$-\pi/6$	$2\sqrt{3}/3$	
0	1	Abs. min
$\pi/3$	2	Abs max