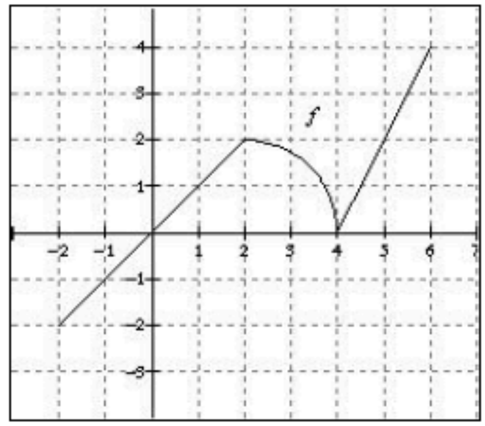
AP Calculus
2nd Fundamental Theorem/Accumulation Functions



1. Let $F(x)=\int_2^x f(t)\,dt$. The graph of *f* on the interval $[-2,6]$ consists of two line segments and a quarter of a circle, as shown at right.

(a) Find $F(0)$ and $F(4)$.

(b) Determine the interval where $F(x)$ is increasing. Justify your answer.

(c) Find the critical numbers of $F(x)$ and determine if each corresponds to a relative minimum value, a relative maximum value, or neither. Justify your answers.

(d) Find the absolute extreme values of $F(x)$ and the *x*-values at which they occur. Justify your answers.

(e) Find the *x*-coordinates of the inflection points of $F(x)$. Justify your answer.

(f) Determine the intervals where the graph of $F(x)$ is concave down. Justify your answer.

2)

Evaluate each expression.

a. $\frac{d}{dx}\left[\int_{1}^{x}\sqrt{t^2-1}\,dt\right]$

b. $\frac{d}{dx}\left[\int_{x}^{3} t\sin t\,dt\right]$

c. $\frac{d}{dx}\left[\int_{\pi/2}^{x^2}\cos t\,dt\right]$

3)

Use the Second Fundamental Theorem of Calculus to find the derivatives of the following functions.

a) $f(x)=\int_{1}^{x}\left(t^2+1\right)^{20}dt$

b) $g(x)=\int_{-1}^{x}\sqrt{t^3+1}\,dt$

c) $g(x)=\int_{\pi}^{x}\frac{1}{1+t^4}\,dt$

d) $f(x)=\int_{4}^{x^2}\cos\left(t^2\right)dt$

4)

Find the interval on which the curve $y=\int_{0}^{x}\left(t^3+t^2+1\right)dt$ is concave up. Justify your answer.

Answers

1. (a) $F(0)=\int_2^0 f(t)\,dt=-\int_0^2 f(t)\,dt=-2$

   $F(4)=\int_2^4 f(t)\,dt=\pi$

   (b) $F(x)$ is increasing when $F'(x)$ is positive.
   Since $F'(x)=f(x)$ and $f(x)$ is positive on the intervals (0,4) and (4,6), then $F(x)$ is increasing on these intervals and in fact on the interval [0,6].

   (c) At $x=0$, $F(x)$ has a relative minimum because $F'=f$ changes from negative to positive there.
   At $x=4$, $F(x)$ has neither a relative minimum nor a relative maximum because $F'=f$ does not change sign there.

   (d) Using the Candidates Test, we compare $F(-2)=0$, $F(0)=-2$, $F(4)=\pi$, and $F(6)=\pi+4$, and find that $F(0)=-2$ is the absolute minimum value of F on the interval $[-2,6]$ and $F(6)=\pi+4$ is the absolute maximum value of F on the interval $[-2,6]$.

   (e) $F(x)$ has inflection points at $x=2$ and $x=4$ because $F''=f'$ changes signs at these points.

   (f) The graph of $F(x)$ is concave down on the interval (2, 4) because $F'(x)=f(x)$ is strictly decreasing on this interval and $F''(x)=f'(x)$ is defined there, or, equivalently, because $F''(x)$ is negative on the interval.

2)

a. $\sqrt{x^2-1}$ b. $-x\sin x$ c. $2x\cos x^2$

3)

a. $f'(x)=(x^2+1)^{20}$ b. $g'(x)=\sqrt{x^3+1}$ c. $g'(x)=\dfrac{1}{1+x^4}$

d. $f'(x)=2x\cos x^4$

4) The function is concave up on the intervals $(-\infty,-2/3)\cup(0,\infty)$ because y'' > 0 on these intervals.