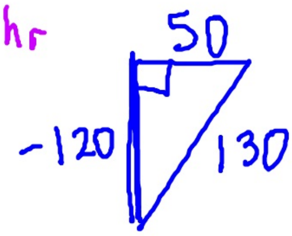
2.6 More Related Rates

1 Two cars start moving from the same point. One travels south at 60 mi/h and the other travels east at 25 mi/hr. At what rate is the distance between the cars increasing two hours later?

$\frac{dy}{dt} = -60$ mi/hr $\quad \frac{dx}{dt} = 25$ mi/hr

$\frac{dc}{dt} = \_\_\_$ when $t = 2$ hours



$$\frac{d}{dt}\left(x^2 + y^2 = c^2\right)$$

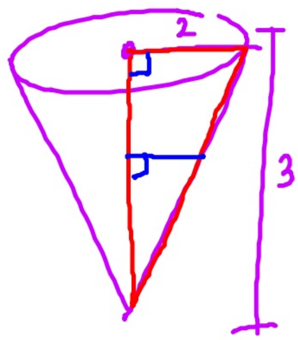
$$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt}$$

$$50(25) + (-120)(-60) = 130\frac{dc}{dt}$$

$$\frac{1250 + 7200}{130} \text{ mi/hr} = \frac{dc}{dt}$$

$$\frac{845}{13} = 65 \text{ mi/hr}$$

#2 A conical tank has height 3 m and radius 2 m at the top.
Water flows in at a rate of 2 cubic meters per minute.
How fast is the water level rising when the height is 2m?



$$\frac{dV}{dt} = 2 \ m^3/min \qquad \frac{dh}{dt} = \_\_ \text{ when } h = 2m$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{r}{h} = \frac{2}{3}$$

$$r = \frac{2}{3}h$$

$$V = \frac{\pi}{3}\left(\frac{2}{3}h\right)^2 h$$

$$V = \frac{4\pi}{27}h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{9}h^2\frac{dh}{dt}$$

$$2 = \frac{4\pi}{9}(2)^2\frac{dh}{dt}$$

$$2 = \frac{16\pi}{9}\frac{dh}{dt}$$

$$\frac{18}{16\pi} = \frac{dh}{dt}$$

m/min

$$\frac{dh}{dt} = \frac{9}{(8\pi)} \text{ m/min}$$

#3 The radius of a right circular cylinder is increasing at a rate of 2 *in/min* and the height is decreasing at a rate of 3 *in/min.* At what rate is the volume changing when the radius is 8 *in* and the height is 12 *in*? Is the volume increasing or decreasing?

$\frac{dr}{dt} = 2 \text{ in/min}$ $\qquad \frac{dV}{dt} = \_\_\_ \qquad r = 8, h = 12$

$\frac{dh}{dt} = -3 \text{ in/min}$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi \left(r^2 \frac{dh}{dt} + h \cdot 2r \frac{dr}{dt} \right)$$

$$= \pi \left(64(-3) + 96(2)(2) \right)$$

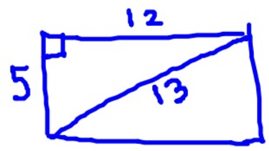
$$= 192\pi \text{ in}^3/\text{min}$$

#4

The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12 cm and the width is 5 cm, find the rates of change of:

a) the area
b) the perimeter
c) the length of a diagonal of the rectangle

$$\frac{dL}{dt} = -2 \text{ cm/sec} \quad \frac{dw}{dt} = 2 \text{ cm/sec}$$



a.) $A = Lw$

$$\frac{dA}{dt} = L\frac{dw}{dt} + w\frac{dL}{dt}$$

$$= 14 \text{ cm}^2/\text{sec}$$

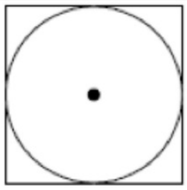
b.) $P = 2L + 2W$

$$\frac{dP}{dt} = 2\frac{dL}{dt} + 2\frac{dw}{dt}$$

$$= 0 \text{ cm/sec}$$

c.) $L^2 + W^2 = C^2$

$$\frac{dC}{dt} = -\frac{14}{13} \text{ cm/sec}$$



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle **expands**, the square **expands** to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.