49.) $y^2 = x^3$

$$2yy' = 3x^2$$

$$y' = \frac{3}{2} \cdot \frac{x^2}{y}$$

$$y'' = \frac{3}{2}\left(\frac{y \cdot 2x - x^2 \cdot 1 \cdot y'}{y^2}\right)$$

$$= \frac{3}{2}\left(\frac{2xy - x^2\left(\frac{3}{2}\frac{x^2}{y}\right)}{y^2}\right)$$

$$= \frac{3}{2}\left(\frac{4xy^2 - 3x^4}{2y^3}\right)$$

$$= \frac{3}{2}\left(\frac{x(4y^2 - 3x^3)}{2y^3}\right)$$

$$\frac{3}{2}\left(\frac{x(4x^3 - 3x^3)}{2y^3}\right)$$

$$\frac{3}{2} \cdot \frac{x^4}{2y^3}$$

$$\frac{3x^4}{4y^3}$$

2.6 Related Rates

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- Find a related rate.
- Use related rates to solve real-life problems.

GUIDELINES FOR SOLVING RELATED-RATE PROBLEMS

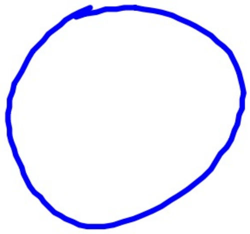
1. Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time t*.
4. *After* completing Step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

$$\frac{d}{dt}\left(x^2 + y^3 = 4x + y\right)$$

$$2x\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 4\frac{dx}{dt} + 1\cdot\frac{dy}{dt}$$

#1 The radius of a circular oil slick expands at a rate of 2 m/min.

How fast is the area of the oil slick increasing when the radius is 25 m?



$$\frac{dr}{dt} = 2 \text{ m/min} \qquad \left.\frac{dA}{dt}\right|_{r=25m} = \underline{\qquad\qquad}$$

$$\frac{d}{dt}\left(A = \pi r^2\right)$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(25)(2) = \boxed{100\pi \text{ m}^2/\text{min}}$$

You must include units with your answer.

#2 A sphere's volume is changing at a rate of 14 in^3/min. Determine the rate at which the radius is changing when the volume is 32pi cubic inches.

$\frac{dV}{dt} = 14$ in^3/min $\quad \frac{dr}{dt} = \_\_\_$ when $V = 32\pi$ in^3

$\frac{d}{dt}\left(V = \frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$r = \sqrt[3]{24}$

$14 = 4\pi r^2 \frac{dr}{dt}$

$14 = 4\pi(\sqrt[3]{24})^2 \frac{dr}{dt}$

$\boxed{\frac{14}{4\pi \cdot 24^{2/3}} = \frac{dr}{dt} \text{ in/min}}$

#3

All edges of a cube are expanding at a rate of 6 cm/sec.

$V = s^3$

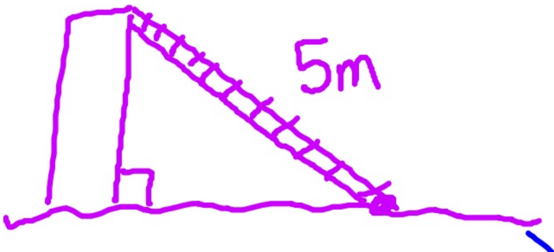
How fast is the volume changing when each edge is 2cm?

$\frac{de}{dt} = 6$ cm/sec $\quad \frac{dV}{dt} = \_\_\_$ when $e = 2$ cm

$$\frac{d}{dt}\left(V = e^3\right)$$

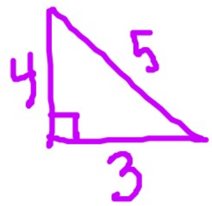
$$\frac{dV}{dt} = 3e^2 \frac{de}{dt} = 3(2)^2 6 = 72 \text{ cm}^3/\text{sec}$$

#4 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. How fast is the top of the ladder descending when the foot of the ladder is 3 m from the house?



$$\frac{dx}{dt} = 0.4 \text{ m/sec}$$

$$\frac{dy}{dt} = \_\_ \text{ when } x = 3\text{m}$$



$$\frac{d}{dt}\left(x^2 + y^2 = c^2\right)$$

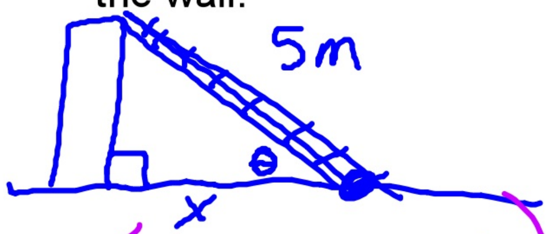
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

$$3(.4) + 4\left(\frac{dy}{dt}\right) = 0$$

$$\frac{dy}{dt} = -\frac{3}{10} \text{ m/sec}$$

#5 A 5 meter long ladder is leaning against the side of a house. The foot of the ladder is pulled away from the house at the rate of 0.4 m/sec. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 4 meters from the wall.



$\frac{dx}{dt} = 0.4$ m/sec

$\frac{d\theta}{dt} = \_\_\_$ when $x = 4$m

$$\frac{d}{dt}\left(\cos\theta = \frac{x}{5}\right)$$

$$-\sin\theta \cdot \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt}$$

$$-\frac{3}{5} \cdot \frac{d\theta}{dt} = \frac{1}{5}(.4)$$

$$\frac{d\theta}{dt} = \frac{2}{25}\left(\frac{-5}{3}\right)$$

$$\frac{d\theta}{dt} = \frac{-2}{15} \text{ rad/sec}$$