2.4 The Chain Rule

- **Find the derivative of a composite function using the Chain Rule.**
- **Find the derivative of a function using the General Power Rule.**
- **Simplify the derivative of a function using algebra.**
- **Find the derivative of a trigonometric function using the Chain Rule.**

We've taken a lot of derivatives over the course of the last few sections. However, if you look back they have all been functions similar to the following kinds of functions.

#1 $$y = \frac{x}{\sqrt{x^4 + 4}}$$

Find y'
Differentiate
Find dy/dx

$$y = x\left(x^4 + 4\right)^{-1/2}$$

$$y' = \underbrace{x \cdot \left(\frac{-1}{2}\left(x^4 + 4\right)^{-3/2} \cdot 4x^3\right)} + \left(x^4 + 4\right)^{-1/2} \cdot 1$$

$$y' = \underbrace{-2x^4\left(x^4 + 4\right)^{-3/2}} + \underbrace{\left(x^4 + 4\right)^{-1/2}}$$

$\frac{-1}{2} - \frac{-3}{2}$

$$y' = \left(x^4 + 4\right)^{-3/2}\left(-2x^4 + \left(x^4 + 4\right)^{1}\right) = \frac{-x^4 + 4}{\left(x^4 + 4\right)^{3/2}}$$

#2

$g(\theta) = \cos^2 8\theta$

$y = \cos 8\theta$

$y' = \underbrace{-\sin 8\theta \cdot 8}$

$g(\theta) = (\cos 8\theta)^2$

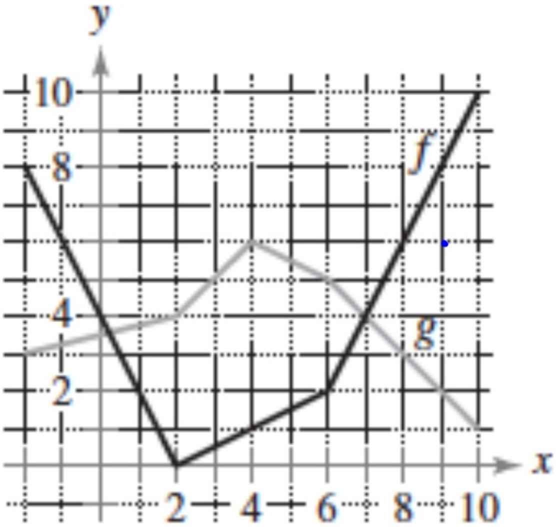
$g'(\theta) = 2(\cos 8\theta)^1 \cdot \underbrace{(-\sin 8\theta) \cdot 8}$

$= -16 \cos 8\theta \sin 8\theta$

$g'\left(\frac{\pi}{12}\right) = -16 \cos\left(\frac{8\pi}{12}\right) \sin\left(\frac{8\pi}{12}\right)$

$= -16\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$

#3 In Exercises 109 and 110, the graphs of f and g are shown. Let $h(x) = f(g(x))$ and $s(x) = g(f(x))$. Find each derivative, if it exists. If the derivative does not exist, explain why.



(a) Find $h'(3)$.

(b) Find $s'(9)$.

a.) $h'(x) = f'(g(x)) \cdot g'(x)$

$h'(3) = f'(g(3)) \cdot g'(3)$

$= f'(5) \cdot 1$

$= \frac{1}{2}$

b.) $s'(9) = g'(f(9)) \cdot f'(9)$

$= g'(8) \cdot 2$

$= -2$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

#4 $h(x) = 3(g(x))^3$; Find $h'(2)$

$$h'(x) = 3\left(3(g(x))^2 \cdot g'(x)\right)$$

$$h'(2) = 9\left(g(2)\right)^2 g'(2)$$

$$= 9\,(3)^2\,(1)$$

$$= 81$$

#5 $t(x)=f(x)/g(x)$; Find $t'(2)$

$$t'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$$

$$= \frac{(3)(2) - (9)(1)}{9}$$

$$= \frac{-1}{3}$$

#6 $p(x) = f(g(x))$; Find $p'(2)$

$$p'(2) = f'(g(2)) \cdot g'(2)$$

$$= f'(3) \cdot (1)$$

$$= -4$$

Product rule
Quotient rule
All trig derivatives
Chain rule
charts and graphs
differentiability
equations of tangent lines
equations of normal lines
horizontal tangents
slope
second derivative
derivatives of sinx and cosx (ex. 50th deriv)

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$$y = \cos x$$
$$y' = -\sin x$$
$$y'' = -\cos x$$
$$y''' = \sin x$$
$$y^{(4)} = \cos x$$

$$y^{(27)} = \sin x$$

79a.) $f(x) = \sin \underbrace{2x}$ $(\pi, 0)$

$f'(x) = 2\cos 2x$

$\cos 2x \cdot 2$

$f'(\pi) = 2$

$y - 0 = 2(x - \pi)$

48.) $h(x) = \sec x^2$

$h(x) = \sec(x^2)$

$h'(x) = \sec(x^2)\tan(x^2) \cdot 2x$

55.) $y = 4\sec^2 x$

$\boxed{4\sec x \sec x}$

$= 4(\sec x)^2$

$y' = 8(\sec x)^1 \cdot \sec x \tan x$

$= 8\sec^2 x \tan x$