9.) $f(x) = \dfrac{x^{1/2}}{x^3+1}$

$$f'(x) = \frac{(x^3+1)\frac{1}{2}x^{-1/2} - x^{1/2}(3x^2)}{(x^3+1)^2}$$

$$= \frac{\frac{1}{2}x^{5/2} + \frac{1}{2}x^{-1/2} - 3x^{5/2}}{(x^3+1)^2} = \frac{\frac{-5}{2}x^{5/2} + \frac{1}{2}x^{-1/2}}{(x^3+1)^2}$$

$$= \frac{\frac{-1}{2}x^{-1/2}(5x^3-1)}{(x^3+1)^2} = \frac{-1(5x^3-1)}{2\sqrt{x}(x^3+1)^2}$$

47.) $y = \frac{3(1-\sin x)}{2\cos x} = \frac{3}{2}\left(\frac{1-\sin x}{\cos x}\right)$

$y' = \frac{3}{2}\left(\frac{\cos x(-\cos x)+(1-\sin x)(+\sin x)}{\cos^2 x}\right)$

$= \frac{3}{2}\left(\frac{-\cos^2 x + \sin x - \sin^2 x}{\cos^2 x}\right)$ $\quad -1$

$y' = \frac{3}{2}\left(\frac{\sin x - 1}{\cos^2 x}\right)$

2.4 The Chain Rule

- Find the derivative of a composite function using the Chain Rule.
- Find the derivative of a function using the General Power Rule.
- Simplify the derivative of a function using algebra.
- Find the derivative of a trigonometric function using the Chain Rule.

The Chain Rule is for taking derivatives of composite functions such as:

$h(x) = (3x - 1)^6$ and $s(x) = \sin(5x)$

These functions have an outer and inner function.

Find dy/dx

#1: $y = (2x - 3)^2$

$y = 4x^2 - 12x + 9$

$y' = 8x - 12$

$y' = 4(2x - 3)$

$y = f(g(x))$

$y' = f'(g(x))g'(x)$

$y = (2x-3)^2$

$y' = 2(2x-3)^1 \cdot 2$

$y' = 4(2x-3)$

Differentiate.

#2: $y = (2x - 3)^5$

$$y' = 5(2x-3)^4 \cdot 2$$

$$y' = 10(2x-3)^4$$

#3 Find the derivative.

$$y = \frac{4}{\sqrt{x^2+1}} = 4(x^2+1)^{-1/2}$$

$$y' = -2(x^2+1)^{-3/2} \cdot 2x$$

$$= \frac{-4x}{(x^2+1)^{3/2}}$$

Find an equation of the tangent line to the given point.

#4 $y = \sqrt[5]{3x^3 + 4x}, \quad (2, 2)$

$$y' = \frac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$$

$$y'(2) = \frac{9(2)^2 + 4}{5(3(2)^3 + 4(2))^{4/5}} = \frac{40}{5(32)^{4/5}}$$

$$= \frac{40}{5 \cdot 16} = \frac{8}{16} = \frac{1}{2}$$

$$\boxed{y - 2 = \frac{1}{2}(x - 2)}$$

#5: y = cos(2x)

$y' = -\sin(2x) \cdot 2$

$y' = -2\sin 2x$

$y = (x)^7$

$y' = 7(x)^6 \cdot 1$

$y = \sin(x)$

$y' = \cos(x) \cdot 1$

#6: $y = \sin^3 x$

$$y = (\sin x)^3$$

$$y' = 3(\sin x)^2 \cos x$$

$$y' = 3\sin^2 x \cos x$$

#7: $y = \tan^2 x$

$$y = (\tan x)^2$$

$$y' = 2(\tan x)^1 \sec^2 x$$

8.) $y = \sec^4 x$

$y = (\sec x)^4$

$y' = 4(\sec x)^3 \sec x \tan x$

$= 4\sec^4 x \tan x$

$f(x) = x^5 \qquad g(x) = 7x+1$

$f(g(x)) = (7x+1)^5$