More 2.3: Product and Quotient Rule

#1 $y = \dfrac{1 - \cos x}{\sin x}$

$$y' = \frac{\sin x(\sin x) - (1-\cos x)\cos x}{\sin^2 x}$$

$$y' = \frac{\sin^2 x - \cos x + \cos^2 x}{\sin^2 x}$$

$$y' = \frac{1-\cos x}{\sin^2 x}$$

#2 $h(s) = \dfrac{s}{\sqrt{s}-1}$

$s^{1/2} - 1$

$$h'(s) = \frac{(\sqrt{s}-1)\cdot 1 - s^{1/2}\left(\frac{1}{2}s^{-1/2}\right)}{(\sqrt{s}-1)^2}$$

$$= \frac{s^{1/2} - 1 - \frac{1}{2}s^{1/2}}{(\sqrt{s}-1)^2}$$

$$= \frac{\frac{1}{2}s^{1/2} - \frac{2}{2}}{(\sqrt{s}-1)^2}$$

$$\frac{\frac{s^{1/2}-2}{2}}{(\sqrt{s}-1)^2}$$

$$\frac{\sqrt{s}-2}{2(\sqrt{s}-1)^2} = h'(s)$$

#3 Find the points where f(x) has a horizontal tangent

$$f(x) = \frac{x^2}{x^2 + 1}$$

$$f'(x) = \frac{(x^2+1)2x - x^2(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2x}{(x^2+1)^2}$$

$$0 = \frac{2x}{(x^2+1)^2}$$

$0 = 2x$; $x = 0$

$(0, 0)$

#4

Find the derivative of f(x) = tanx

$$f(x) = \frac{\sin x}{\cos x}$$

$$f'(x) = \frac{\cos x(\cos x) - \sin(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$f(x) = \cot x$$

$$f'(x) = -\csc^2 x$$

$$f(x) = \sec x = \frac{1}{\cos x}$$

$$f'(x) = \frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x}$$

$$f'(x) = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \tan x \sec x$$

$$\frac{d}{dx}[\csc x] = -\cot x \csc x$$

$$\tan x \sec x \sec x$$

$$-\cot x \csc x \csc x$$

$$y = \frac{1-\cos x}{\sin x}$$

$$y = \frac{1}{\sin x} - \frac{\cos x}{\sin x}$$

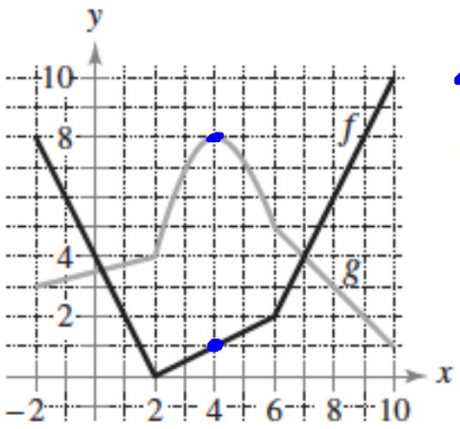
$$y = \csc x - \cot x$$

$$y' = -\cot x \csc x + \csc^2 x$$

$$y' = \frac{-\cos x}{\sin x \cdot \sin x} + \frac{1}{\sin^2 x} = \frac{1-\cos x}{\sin^2 x}$$

#5: Product Rule and Quotient Rule

. (a) Find $p'(4)$.



$$p(x) = \frac{g(x)}{f(x)}$$

$$p'(4) = \frac{f(4)g'(4) - g(4)f'(4)}{(f(4))^2}$$

$$= \frac{(1)(0) - (8)\left(\frac{1}{2}\right)}{1^2}$$

$$= -4$$

#6: Find f''(x)

$$f(x) = \sec x$$

$$f'(x) = \tan x \sec x$$

$$f''(x) = \tan x \tan x \sec x + \sec x \sec^2 x$$

$$= \sec x (\tan^2 x + \sec^2 x)$$

$$= \sec x (\sec^2 x - 1 + \sec^2 x)$$

$$= \sec x (2\sec^2 x - 1)$$

Higher Order Derivatives

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$s(t)$	Position function
$v(t) = s'(t)$	Velocity function
$a(t) = v'(t) = s''(t)$	Acceleration function

First derivative:	y',	$f'(x)$,	$\frac{dy}{dx}$,	rate of change of position
Second derivative:	y'',	$f''(x)$,	$\frac{d^2y}{dx^2}$,	rate of change of velocity
Third derivative:	y''',	$f'''(x)$,	$\frac{d^3y}{dx^3}$,	rate of change of acceleration
Fourth derivative:	$y^{(4)}$,	$f^{(4)}(x)$,	$\frac{d^4y}{dx^4}$,	
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nth derivative:	$y^{(n)}$,	$f^{(n)}(x)$,	$\frac{d^ny}{dx^n}$,	

$y = \sin x$

$y' = \cos x$

$y'' = -\sin x$

$y''' = -\cos x$

$y^{(4)} = \sin x$

*

$y^{(27)} = -\cos x$

$y^{(28)} = \sin x$