2.3 Product and Quotient Rules and Higher-Order Derivatives

- Find the derivative of a function using the Product Rule.
- Find the derivative of a function using the Quotient Rule.
- Find the derivative of a trigonometric function.
- Find a higher-order derivative of a function.

The Product Rule

$$\frac{d}{dx}[fg] = f \cdot g' + g \cdot f'$$

#1

$$h(x) = \overset{f}{(3x - 1)}\overset{g}{(4x + 5)}$$

$h'(x) = (3x-1)4 + (4x+5)\cdot 3$

$= 24x + 11$

$h(x) = 12x^2 + 11x - 5$

$h'(x) = 24x + 11$

#2: Write the equation of the line tangent to f(x) at x = 0

$f(x) = \sin x \cos x$

$(0, 0)$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$

$f'(x) = \cos^2 x - \sin^2 x$

$f'(0) = 1 - 0$

$f'(0) = 1$

$y = x$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx}\left[\frac{Hi}{Lo}\right] = \frac{Lo\,dHi - Hi\,dLo}{Lo^2}$$

Find f'(x) and f'(c)

#3 $f(x) = x \cos x$ $\qquad c = \pi$

$f'(x) = x(-\sin x) + \cos x \cdot 1$

$f'(\pi) = 0 - 1$

$= -1$

Write the equation of the tangent line to the graph of f at the given x-value.

#4

$f(x) = \frac{\sin x}{x}$ $\quad x = \frac{\pi}{2}$

$f'(x) = \frac{x\cos x - \sin x \cdot 1}{x^2}$ $\quad \left(\frac{\pi}{2}, \frac{2}{\pi}\right)$

$f'\left(\frac{\pi}{2}\right) = \frac{0 - 1}{\left(\frac{\pi}{2}\right)^2}$

$= \frac{-4}{\pi^2}$

$y - \frac{2}{\pi} = \frac{-4}{\pi^2}\left(x - \frac{\pi}{2}\right)$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	-1	1	2
2	1	$\frac{1}{2}$	3	0
3	3	2	1	-2

#5

Given $h(x) = f(x) - g(x)$, find $h'(2)$

$h(x) = f(x) - g(x)$

$h'(x) = f'(x) - g'(x)$

$h'(2) = f'(2) - g'(2)$

$= \frac{1}{2} - 0$

$= 1/2$

Given $h(x) = f(x) \cdot g(x)$, find $h'(3)$

$h(x) = f(x) \cdot g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$h'(3) = f(3)g'(3) + g(3)f'(3)$

$h'(3) = 3 \cdot -2 + 1 \cdot 2$

$= -4$

#6 Find the slope of the normal line at x = 2

$f(x) = 4x/(x^2+6)$

$f(x) = \frac{4x}{x^2+6}$

perpendicular

$f'(x) = \frac{(x^2+6)\cdot 4 - 4x\cdot 2x}{(x^2+6)^2}$

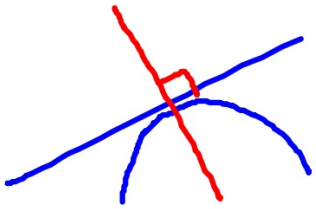
$f'(x) = \frac{-4x^2+24}{(x^2+6)^2}$

$f'(2) = \frac{-16+24}{100}$

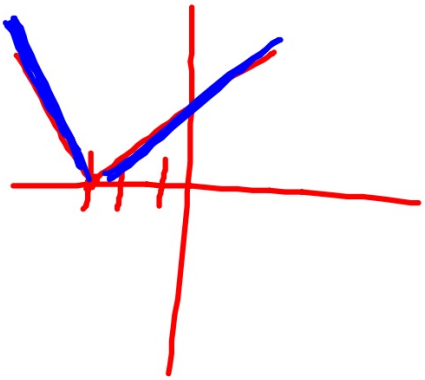
$= \frac{8}{100}$

$-\frac{100}{8}$

$-\frac{25}{2}$



$f(x) = |x+3|$



$$\lim_{x \to -3^-} f'(x) \neq \lim_{x \to -3^+} f'(x)$$

$$-1 \neq 1$$