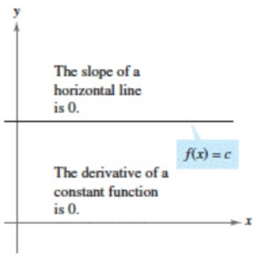
2.2 Basic Differentiation Rules and Rates of Change

- Find the derivative of a function using the Constant Rule.
- Find the derivative of a function using the Power Rule.
- Find the derivative of a function using the Constant Multiple Rule.
- Find the derivative of a function using the Sum and Difference Rules.
- Find the derivatives of the sine function and of the cosine function.
- Use derivatives to find rates of change.



THEOREM 2.2 THE CONSTANT RULE

The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c] = 0.$$

Findf"(x)

#1 $f(x) = x^2$

$f'(x) = 2x$

#2 $f(x) = x^{-5}$

$f'(x) = -5x^{-6}$

$= \frac{-5}{x^6}$

THEOREM 2.3 THE POWER RULE

If n is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}[x^n] = nx^{n-1}.$$

For f to be differentiable at $x = 0$, n must be a number such that x^{n-1} is defined on an interval containing 0.

$6 - x^2$

$-2x^1$

nx^{n-1}

Find y'. Rewrite the function to make it 'derivative ready' if necessary.

#3

$$y = x^{16}$$

$$y' = 16x^{15}$$

#4

$$y = \frac{1}{x^8}$$

$$y = x^{-8}$$

$$y' = -8x^{-9}$$

#5

$$g(x) = \sqrt[4]{x}$$

$$g(x) = x^{1/4}$$

$$g'(x) = \frac{1}{4}x^{-3/4}$$

THEOREM 2.4 THE CONSTANT MULTIPLE RULE

If f is a differentiable function and c is a real number, then cf is also differentiable and $\frac{d}{dx}[cf(x)] = cf'(x)$.

#6

$$f(x) = 2x^3 - x^2 + 3x$$

$$f'(x) = 2 \cdot 3x^2 - 1 \cdot 2x + 3 \cdot 1$$

$$= 6x^2 - 2x + 3$$

#7

$$y = 8x^3 - \frac{2}{x} + 2\sqrt{x}$$

$$y = 8x^3 - 2x^{-1} + 2x^{1/2}$$

$$y' = 24x^2 + 2x^{-2} + 1x^{-1/2}$$

#8

$f(t) = 3 - \frac{3}{5t}$

$f(t) = 3 - \frac{3}{5}t^{-1}$

$f'(t) = 0 + \frac{3}{5}t^{-2}$

$= \frac{3}{5t^2}$

#9

$f(x) = \frac{x^3 - 6}{x^2}$

$f(x) = x - \frac{6}{x^2}$

$f(x) = x - 6x^{-2}$

$f'(x) = 1 + 12x^{-3}$

#10

$f(x) = 3(5 - x)^2$

$f(x) = 3(25 - 10x + x^2)$

$f(x) = 75 - 30x + 3x^2$

$f'(x) = 0 - 30 + 6x$

$= -30 + 6x$

THEOREM 2.6 DERIVATIVES OF SINE AND COSINE FUNCTIONS

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Show Geogebra demonstration...

#11

$f(x) = x + 7 - \sin x$

$f(x) = x + 7 - 1 \cdot \sin x$

$f'(x) = 1 - 1 \cdot \cos x$

$= 1 - \cos x$

#12

$f(x) = 7x + 3\sin x - 6\cos x$

$f'(x) = 7 + 3\cos x + 6\sin x$

#13: Determine if f(x) is differentiable at x = 3.

$$f(x)=\begin{cases} -x+4 & x<3 \\ x^2-8 & x\geq 3 \end{cases}$$

Continuity? yes

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$$

$$1 = 1 \checkmark$$

Differentiability?

$$\lim_{x\to 3^-} f'(x) = \lim_{x\to 3^+} f'(x) = f'(3)$$

$$\lim_{x\to 3^-} (-1) = \lim_{x\to 3^+} (2x)$$

$$-1 \neq 6$$

No f(x) is not diff. at x = 3.

#14

Find a and b so that f(x) is differentiable.

$$f(x)=\begin{cases} ax^2-3x & x<1 \\ bx-2 & x\geq 1 \end{cases}$$

Continuity

$$\lim_{x\to 1^-}(ax^2-3x)=\lim_{x\to 1^+}(bx-2)=f(1)$$

$$a-3=b-2$$

Diff.

$$\lim_{x\to 1^-}(2ax-3)=\lim_{x\to 1^+} b=f'(1)$$

$$2a-3=b$$

$$a-3=(2a-3)-2$$

$$a-3=2a-5$$

$$2=a$$

$$b=1$$