2.1 The Derivative and the Tangent Line Problem

- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.

Alternate Form of the Derivative

(provided the limit exists)

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Note: This is a general limit!!!

Find g'(1) using the alternate form of the derivative

Hint: we did this problem earlier

#1 $g(x) = 6 - x^2$

$g(1) = 5$

$\boxed{g'(1) = -2}$

$$g'(1) = \lim_{x \to 1} \frac{g(x) - g(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{6 - x^2 - 5}{x - 1} = \lim_{x \to 1} \frac{1 - x^2}{x - 1}$$

$$= \lim_{x \to 1} \frac{(1-x)(1+x)}{x - 1} = \lim_{x \to 1} \frac{-\cancel{(x-1)}(1+x)}{\cancel{x-1}}$$

$$= -2$$

$f(x) = 2x^2 + 3x$

$f'(3) =$

Use alternate form of the derivative to find f'(3)

$$f'(3) = \lim_{x\to 3} \frac{2x^2+3x-27}{x-3}$$

$$= \lim_{x\to 3} \frac{(2x+9)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 15$$

Differentiability

A function is considered differentiable at a point if the derivative can be found.

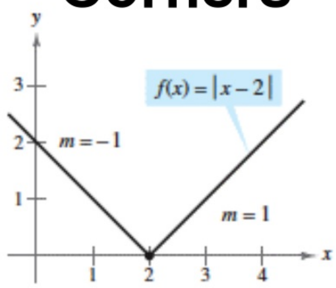
Continuity

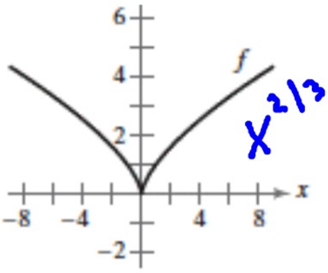
$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)$$

$$\lim_{x \to c^-} f'(x) = \lim_{x \to c^+} f'(x) = f'(c)$$

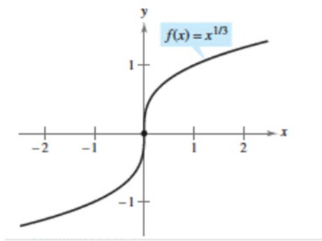
A function is not differentiable when the function has:

Corners **Cusps** **Vertical Tangents**



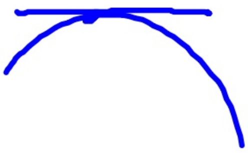


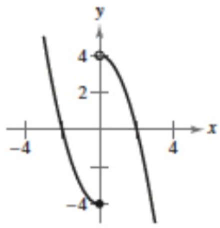
$x^{2/3}$

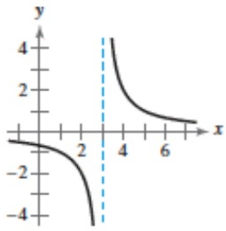


$\sqrt[3]{x}$

any type of discontinuity

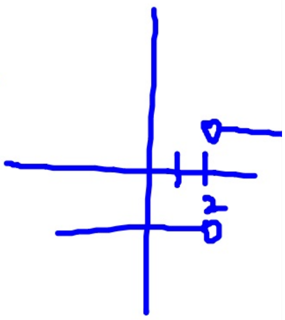






#2 Why are corners not differentiable?

$$\frac{|x-2|}{x-2}$$

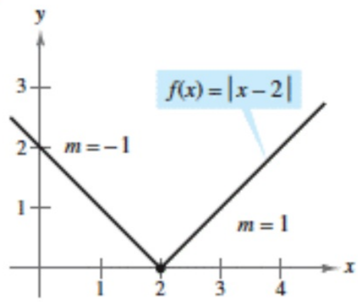


$$f(x) = |x - 2|$$

$$\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$$

$$\lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$$

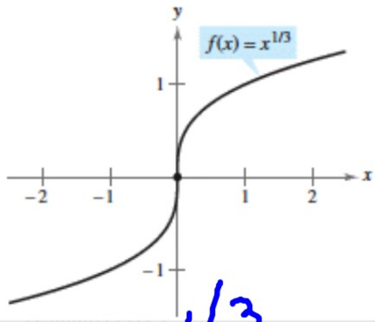
$$\lim_{x\to 2^-} f'(x) \neq \lim_{x \to 2^+} f'(x)$$



f is not differentiable at $x = 2$, because the derivatives from the left and from the right are not equal.

#3 Why are vertical tangents not differentiable?

$$\lim_{x \to 0^-} \frac{x^{1/3} - 0}{x - 0} = \lim_{x \to 0^-} \frac{1}{x^{2/3}} = \infty$$



$$f(x) = x^{1/3}$$

$$\frac{1}{\frac{1}{1000}}$$

$$\frac{1}{(-.01)^{2/3}}$$

$$\frac{1}{\text{small}} \to \text{big}$$

True or false.
Explain.

If f(x) is continuous at x = c,then f(x) is differentiable at x = c

False ; $f(x) = |x|$

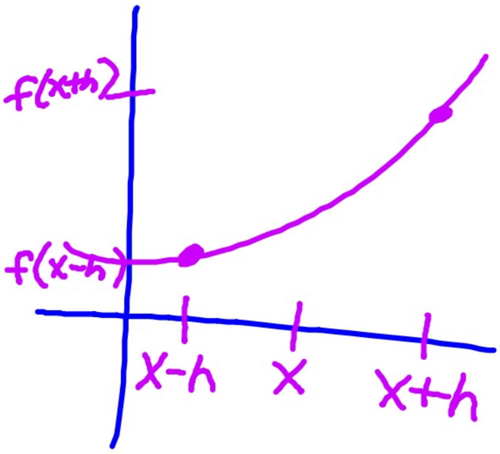
If f(x) is differentiable at x = c, then f(x) is continuous at x = c.

True

Differentiability implies continuity BUT continuity does not imply differentiability!!!!!!!!!!!

Another way to find a derivative:

the symmetric difference quotient



$(x-h, f(x-h))$

$(x+h, f(x+h))$

$$\frac{f(x+h)-f(x-h)}{x+h-(x-h)}$$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h}}$$

$f(x) = \sin x$ $\qquad\qquad f'(x) = \cos x$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x-h)}{2h}$$

$$\lim_{h \to 0} \frac{\cancel{\sin x \cos h} + \sin h \cos x - (\cancel{\sin x \cos h} - \sin h \cos x)}{2h}$$

$$\lim_{h \to 0} \frac{\cancel{2}\sin h \cos x}{\cancel{2}h} = \lim_{h \to 0} \frac{\sin h}{h} \cdot \cos x$$

$$1 \cdot \cos x$$

$$= \cos x$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

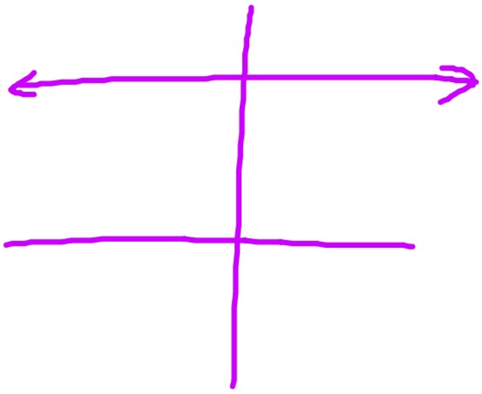
$$\boxed{f'(x) = -\sin x}$$

11.) $f(x) = 7$

$$\lim_{h \to 0} \frac{7-7}{h}$$

$$\lim_{h \to 0^-} \frac{7-7}{h} \qquad \lim_{h \to 0^+} \frac{7-7}{h}$$

$$0 = 0$$



$f'(x) = 0$

19.) $f(x) = x^3 - 12x$

$$\lim_{h \to 0} \frac{(x+h)^3 - 12(x+h) - (x^3 - 12x)}{h}$$

$$(x+h)^3 \neq x^3 + h^3$$