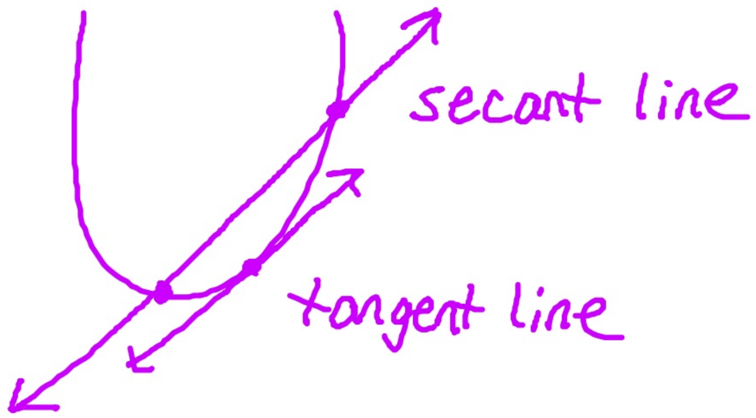
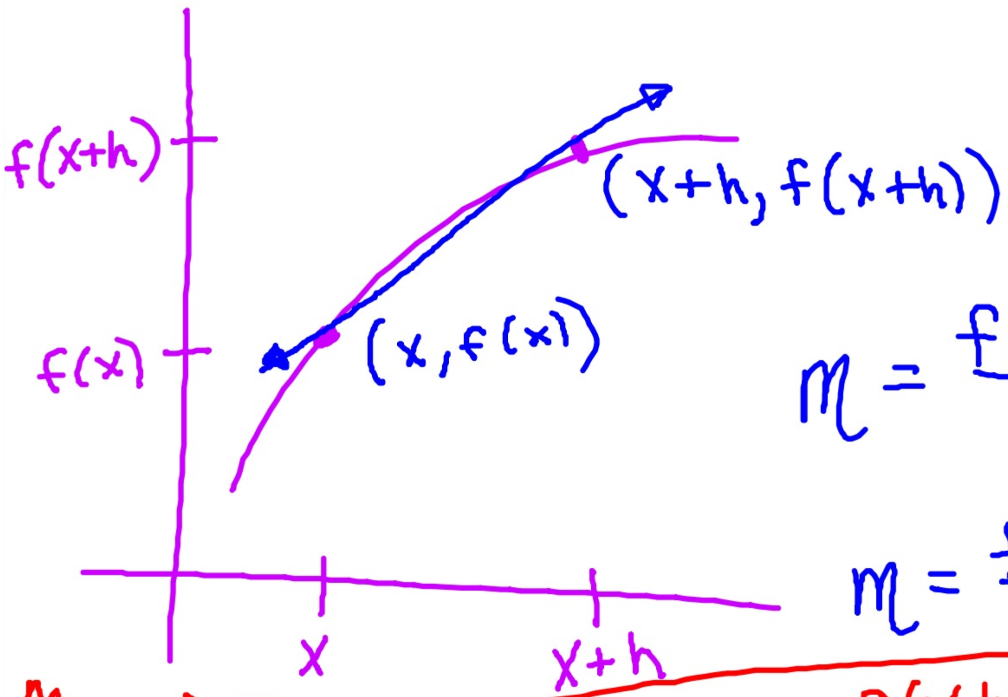
2.1 The Derivative and the Tangent Line Problem

- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{sec} = \frac{f(c + \Delta x) - f(c)}{(c + \Delta x) - c}$$



$\Delta x = h$



$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

$m_{sec} \rightarrow m_{tan}$

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

Synonyms for derivatives

slope
rate of change
slope of tangent

Find the derivative by the limit process.

$f(x+h) = 3(x+h)+2$

#1 $f(x) = 3x + 2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h)+2-(3x+2)}{h}$$

$$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}}$$

$$f'(x) = 3$$

Ways to express derivatives

$$\boxed{f'(x), \quad \frac{dy}{dx}, \quad y', \quad \frac{d}{dx}[f(x)],} \quad \cancel{D_x[y]}.$$

most common

not on AP exam

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= f'(x).$$

#2 $g(x) = 6 - x^2$

$$g(x+h) = 6 - (x+h)^2$$

$$g'(x) = \lim_{h\to 0} \frac{6-(x+h)^2 - (6-x^2)}{h}$$

$$= \lim_{h\to 0} \frac{6-(x^2+2xh+h^2)-6+x^2}{h}$$

$$= \lim_{h\to 0} \frac{-2xh-h^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = -2x$$

$$\boxed{g'(x) = -2x}$$

#3 $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

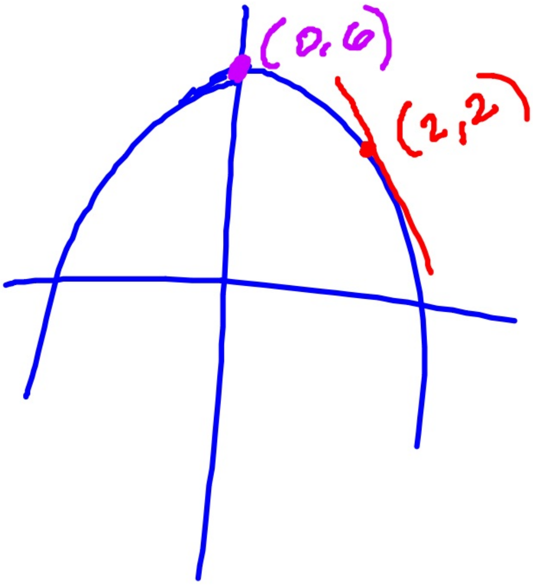
$$= \lim_{h \to 0} \frac{\cancel{x} + \cancel{h} - \cancel{x}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

#2 $g(x) = 6 - x^2$

$g'(x) = -2x$



$g'(0) = 0$

slope of g at $x=0$ is 0.

$g'(2) = -4$

#4

$h(x) = \frac{1}{x}$

LCD: $x(x+h)$

$$h'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h}\,x(x+h) - \frac{1}{x}\,x(x+h)}{h\,x(x+h)}$$

$$= \lim_{h \to 0} \frac{x-(x+h)}{xh(x+h)}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x^2}$$

Synonyms for derivative

Slope
Rate of Change
Velocity
Rise/Run

Another way to find the derivative (symmetric difference quotient

$$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{x+h-(x-h)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$g(x) = 6 - x^2$$

$$g(x+h) = 6 - (x+h)^2$$

$$g(x-h) = 6 - (x-h)^2$$

$$g'(x) = \lim_{h \to 0} \frac{6-(x+h)^2 - \left(6-(x-h)^2\right)}{2h}$$

$= -2x$ (the same answer as example 2)