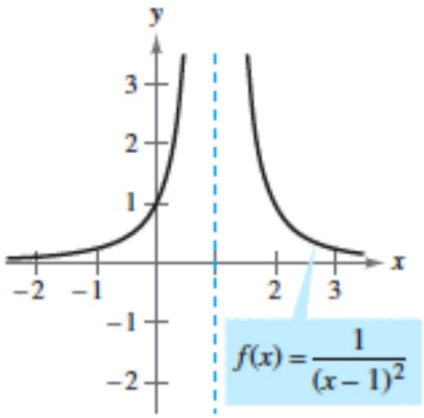
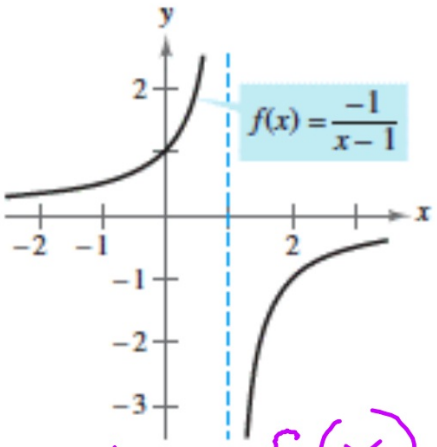
1.5 Infinite Limits

- Determine infinite limits from the left and from the right.
- Find and sketch the vertical asymptotes of the graph of a function.



$\lim_{x \to 1} f(x) = \infty$



$\lim_{x \to 1} f(x)$ DNE

DEFINITION OF VERTICAL ASYMPTOTE

If $f(x)$ approaches infinity (or negative infinity) as x approaches c from the right or the left, then the line $x = c$ is a **vertical asymptote** of the graph of f.

Determine all vertical asymptotes of the graph of each function.

a. $f(x) = \dfrac{1}{2(x+1)}$

VA $x = -1$

$\lim_{x \to -1^-} f(x)$

$-\infty$

b. $f(x) = \dfrac{x^2+1}{x^2-1}$

VA $x = \pm 1$

$\lim_{x \to 1^+} f(x)$

$+\infty$

$\lim_{x \to -1^+} f(x)$

$-\infty$

c. $f(x) = \cot x$

$x = 0, \pi, 2\pi, \ldots$

VA $x = \pi n$, $n \in \mathbb{Z}$

$\lim_{x \to \pi^+} \cot x$

$+\infty$

Find all vertical asymptotes.

Ex 1

$$h(t) = \frac{t^2 - 2t}{t^4 - 16}$$

$$= \frac{t(t-2)}{(t^2+4)(t-2)(t+2)}$$

VA: $t = -2$

Find all vertical asymptotes.

Ex 2

$$h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2}$$

$$= \frac{(x+2)(x-2)}{(x^2+1)(x+2)}$$

No VA

THEOREM 1.15 PROPERTIES OF INFINITE LIMITS

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x \to c} f(x) = \infty \quad \text{and} \quad \lim_{x \to c} g(x) = L.$$

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1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x \to c} [f(x)g(x)] = -\infty, \quad L < 0$
3. Quotient: $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

$$\frac{\text{constant}}{\text{Big}} = 0$$

When a limit is approaching a vertical asymptote, there are only 3 possible answers:

infinity

negative infinity

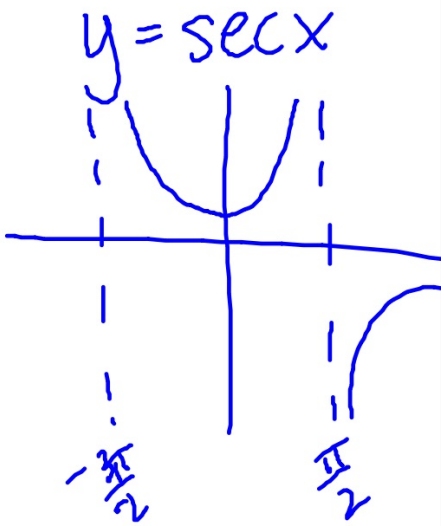
does not exist

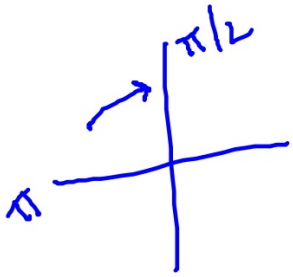
Ex 3

$$\lim_{x \to (\pi/2)^+} \frac{-2}{\cos x}$$

$$\lim_{x \to \frac{\pi}{2}^+} -2(\sec x)$$

$$-2 \lim_{x \to \frac{\pi}{2}^+} \sec x = -2(-\infty) = \infty$$





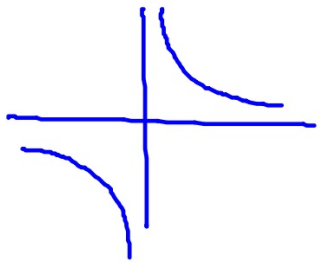
Ex 4

$$\lim_{x \to 0^-} \left(x^2 - \frac{1}{x} \right)$$

$$\lim_{x \to 0^-} x^2 - \lim_{x \to 0^-} \frac{1}{x}$$

$$0 - (-\infty)$$

$$\infty$$



$\frac{1}{-.001}$

$\frac{1}{-\text{small}}$

$-\text{Big}$

$-\infty$