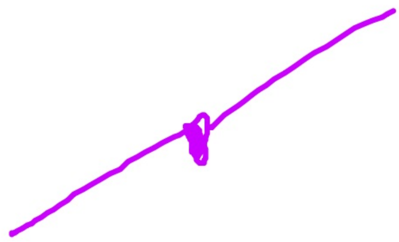
1.4 Continuity and One-Sided Limits

- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

Ex 1 **What value of a will make f(x) continuous?**

$$f(x) = \begin{cases} 3x^3, & x \le 1 \\ ax + 5, & x > 1 \end{cases}$$



$$\lim_{x \to 1^-} 3x^3 = \lim_{x \to 1^+} (ax+5) = f(1)$$

$$3 = a + 5$$

$$-2 = a$$

Ex 2

$$g(x) = \begin{cases} x^2 - c & x < 5 \\ 4x + 2c & x \geq 5 \end{cases}$$

$20 + \frac{10}{3}$

$$\lim_{x \to 5^-} (x^2 - c) = \lim_{x \to 5^+} (4x + 2c) = g(5)$$

$$25 - c = 20 + 2c = \frac{70}{3}$$

$$5 = 3c$$

$$5/3 = c$$

Ex 3*

$$f(x) = \begin{cases} Ax - B\ , & \text{if } x \le -1 \\ 2x^2 + 3Ax + B\ , & \text{if } -1 < x \le 1 \\ 4\ , & \text{if } x > 1 \end{cases}$$

$$\lim_{x \to -1^-} (Ax - B) = \lim_{x \to -1^+} (2x^2 + 3Ax + B) = f(-1)$$

$$\boxed{-A - B = 2 - 3A + B}$$

$$\lim_{x \to 1^-} (2x^2 + 3Ax + B) = \lim_{x \to 1^+} 4 = f(1)$$

$$\boxed{2 + 3A + B = 4}$$

Use elimination (or subst.)

$\cancel{2A - 2B = 2}$

$A - B = 1$

$3A + B = 2$

$4A = 3$

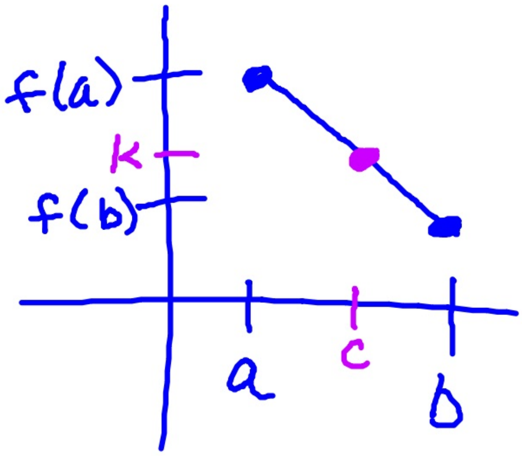
$A = 3/4$

$B = \frac{-1}{4}$

REM 1.13 INTERMEDIATE VALUE THEOREM

ontinuous on the closed interval $[a, b]$, $f(a) \neq f(b)$, and k is any r between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ at

$$) = k.$$

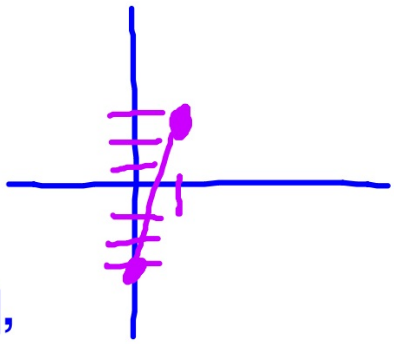


Use IVT to prove that there will be a value c on the closed interval such that f(c) = 0

$$f(x) = x^3 + 5x - 3 \qquad [0, 1]$$

$f(0) = -3$

$f(1) = 3$



Since f(x) is continuous on [0,1], and f(0) < 0 < f(1), by IVT there must exist a value c such that f(c) = 0.

Prove that there is a value c guaranteed by IVT. Then, find c.

$$f(x) = \frac{x^2 + x}{x - 1}, \quad \left[\frac{5}{2}, 4\right], \quad f(c) = 6$$

$f\left(\frac{5}{2}\right) = \frac{35}{6} = 5\frac{5}{6}$

$f(4) = \frac{20}{3} = 6\frac{2}{3}$

Since f(x) is continuous on [2.5, 4] and f(2.5) < 6 < f(4), by IVT there exists a value c such that f(c) = 6.

$\boxed{c = 3}$

$$6 = \frac{c^2 + c}{c - 1}$$

$6c - 6 = c^2 + c$

$c^2 - 5c + 6 = 0$

$(c-2)(c-3) = 0$

$c = \cancel{2}, 3$