1.4 Continuity and One-Sided Limits

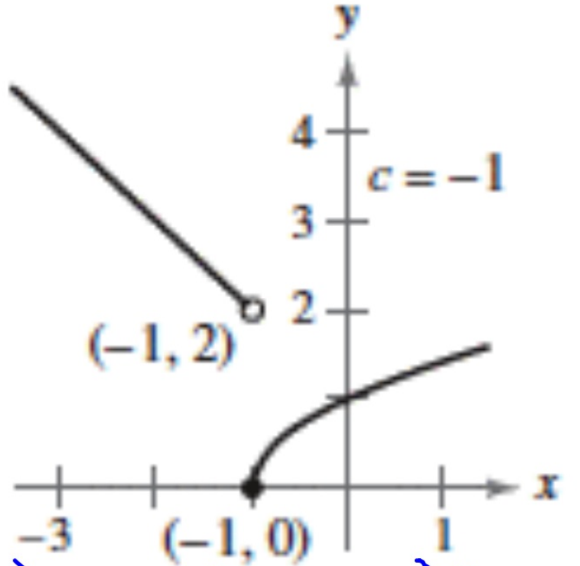
- Determine continuity at a point and continuity on an open interval.
- Determine one-sided limits and continuity on a closed interval.
- Use properties of continuity.
- Understand and use the Intermediate Value Theorem.

One-sided Limits

Evaluate.

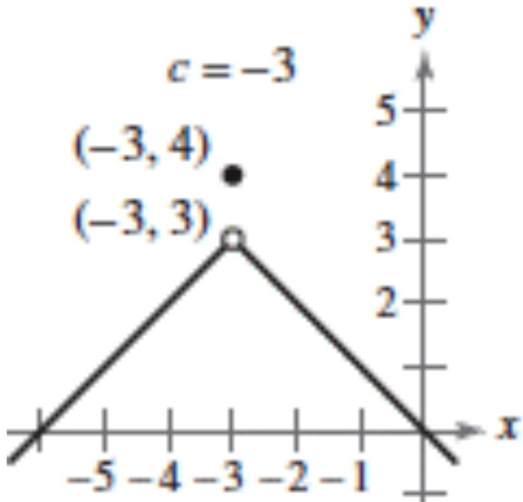
(a) $\lim_{x \to c^+} f(x)$ (b) $\lim_{x \to c^-} f(x)$ (c) $\lim_{x \to c} f(x)$

Ex 1



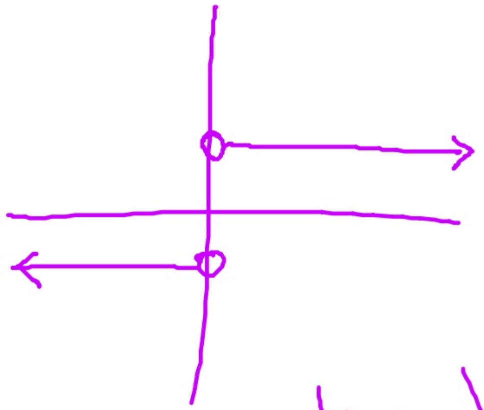
a.) 0 b.) 2 c.) DNE

Ex 2



a.) 3 b.) 3 c.) 3

$$y = \frac{|x|}{x}$$



x	y
-2	-1
-1	-1
0	
1	1
2	1

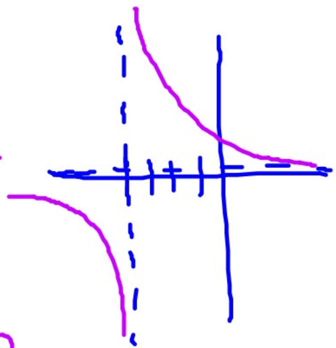
$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x \to 0^+} \frac{|x|}{x} = 1$$

Ex 3

$$\lim_{x\to -4^+} \frac{x-4}{x^2-16} = \lim_{x\to -4^+} \frac{\cancel{x-4}}{(x+4)\cancel{(x-4)}}$$

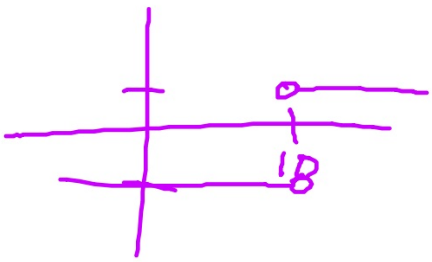
$(x+4)(x-4)$



$$= \lim_{x\to -4^+} \frac{1}{x+4} \quad \infty$$

Ex 4

* $$\lim_{x\to 10^+} \frac{|x-10|}{x-10} = 1$$



$$\lim_{x \to 7^-} \frac{|7-x|}{7-x}$$

Ex 5

$$\lim_{x\to 2} f(x) = 2, \text{ where } f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \geq 2 \end{cases}$$

$\lim_{x\to 2^-} f(x) = 2$ $\quad$ $\lim_{x\to 2^+} f(x) = 2$

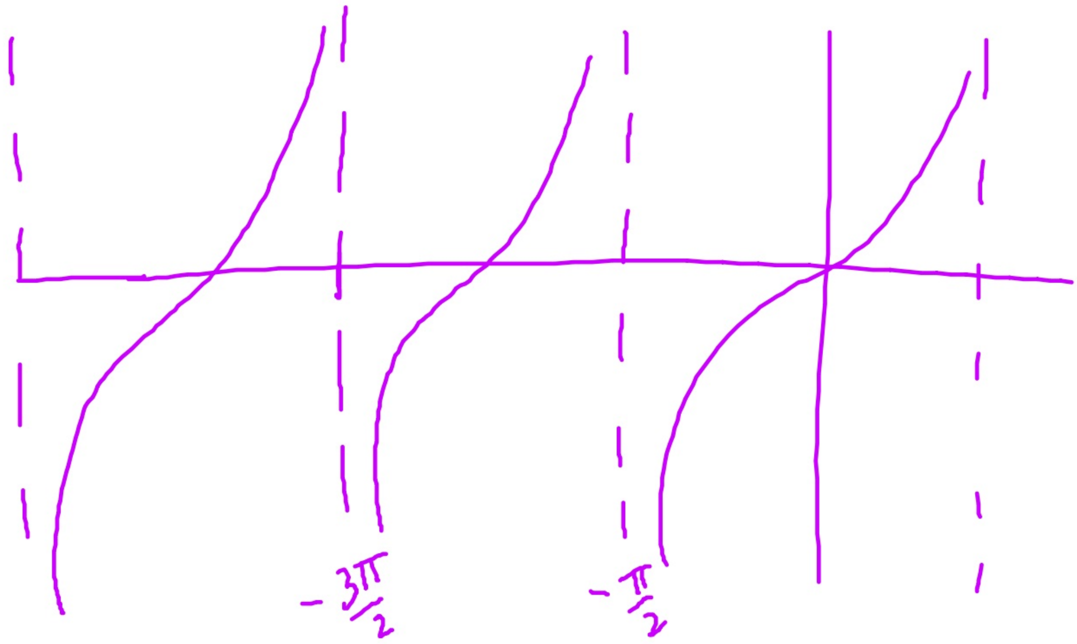
Ex 6

$$\lim_{x\to 2^+} (2x - [\![x]\!]) = \lim_{x\to 2^+} 2x - \lim_{x\to 2^+} [x]$$

$$4 - 2 = 2$$

$\lim_{x \to -3^-} \tan \frac{\pi x}{2}$ $\quad \tan\left(\frac{-3\pi}{2}\right)$

∞ $\quad$ (left)



1.4 Continuity and One-Sided Limits

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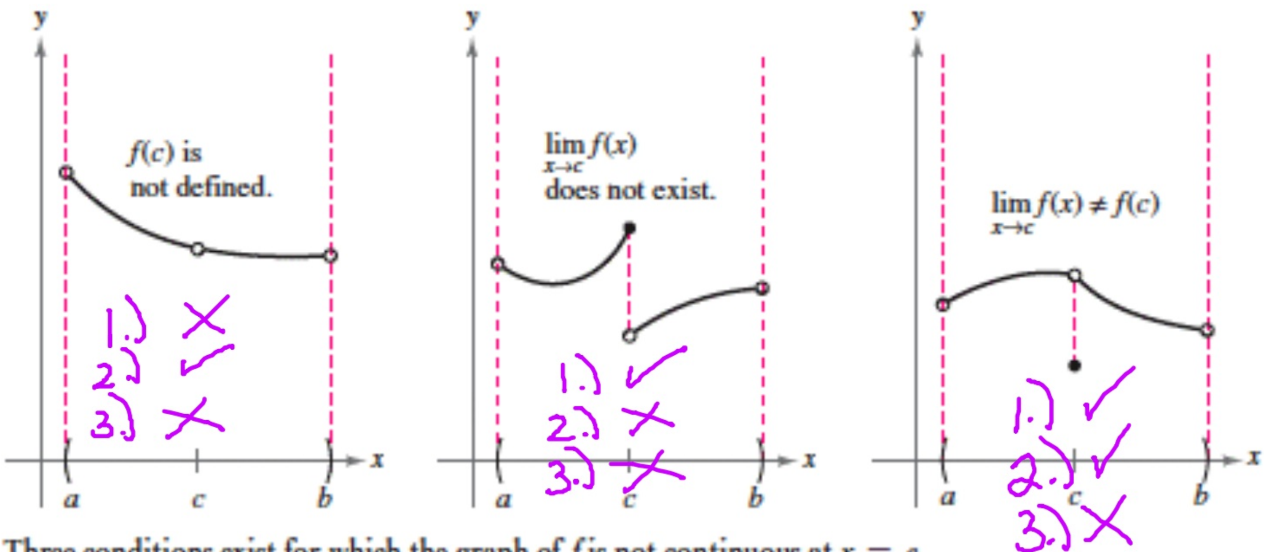
DEFINITION OF CONTINUITY

Continuity at a Point: A function f is **continuous at c** if the following three conditions are met.

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$

Continuity on an Open Interval: A function is **continuous on an open interval (a, b)** if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is **everywhere continuous.**



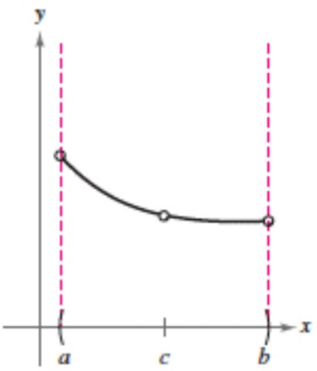
Three conditions exist for which the graph of f is not continuous at $x = c$.

In Figure 1.25, it appears that continuity at $x = c$ can be destroyed by any one of the following conditions.

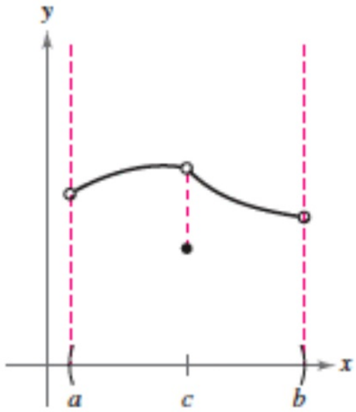
1. The function is not defined at $x = c$.
2. The limit of $f(x)$ does not exist at $x = c$.
3. The limit of $f(x)$ exists at $x = c$, but it is not equal to $f(c)$.

Removable Discontinuity:

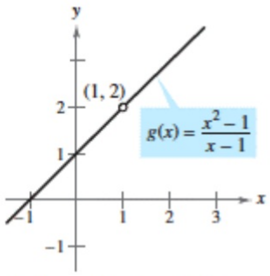
If moving one point can 'fix' the discontinuity, this is called removable.



(a) Removable discontinuity



(c) Removable discontinuity



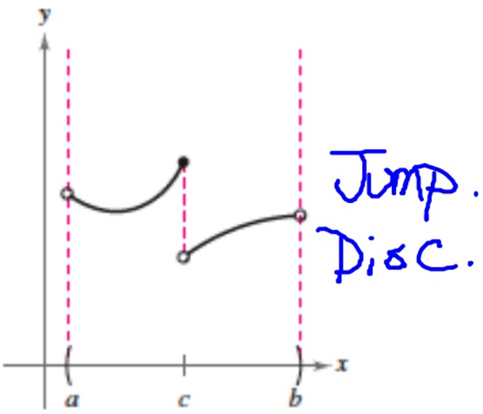
(b) Removable discontinuity at $x = 1$

$\longleftarrow \; g(x) = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

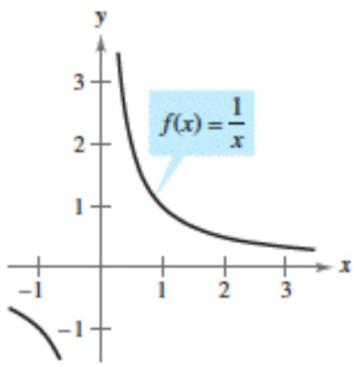
$g(x) = x + 1$

Nonremovable discontinuity:

If 'moving' more than one point is necessary to fix the discontinuity



(b) Nonremovable discontinuity



(a) Nonremovable discontinuity at $x = 0$

THEOREM 1.11 PROPERTIES OF CONTINUITY

If b is a real number and f and g are continuous at $x = c$, then the following functions are also continuous at c.

1. Scalar multiple: bf
2. Sum or difference: $f \pm g$
3. Product: fg
4. Quotient: $\frac{f}{g}$, if $g(c) \neq 0$

The following types of functions are continuous at every point in their domains.

1. Polynomial: $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$
2. Rational: $r(x) = \frac{p(x)}{q(x)}$, $q(x) \neq 0$
3. Radical: $f(x) = \sqrt[n]{x}$
4. Trigonometric: $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, $\csc x$

Discuss the continuity.

Ex 7

$$f(x) = \frac{x-6}{x^2-36}$$

$$f(x) = \frac{\cancel{x-6}}{\cancel{(x-6)}(x+6)}$$

$$f(x) = \frac{1}{x+6}$$

hole @ $\left(6, \frac{1}{12}\right)$

Removable discontinuity at x = 6

$\lim_{x \to 6} f(x)$ exists but $\lim_{x \to 6} f(x) \neq f(6)$

Nonremovable discontinuity at x = -6

$\lim_{x \to -6} f(x)$ dne

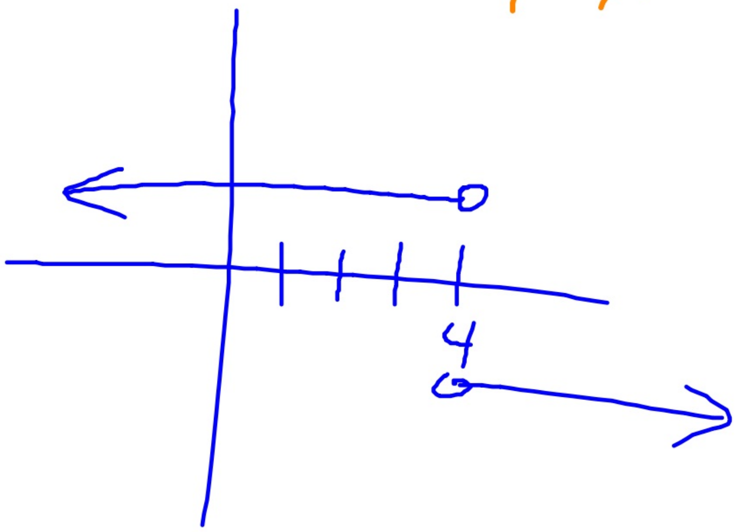
8 $g(x) = 7x - \cos 2x$

$g(x)$ is continuous on $(-\infty, \infty)$

for all values of c

$$\lim_{x \to c} g(x) = g(c)$$

⑨ $g(x) = \frac{|4-x|}{4-x}$



nonremovable disc @ $x=4$

$\lim_{x \to 4} g(x)$ DNE