1.3 Evaluating Limits Analytically

- Evaluate a limit using properties of limits.
- Develop and use a strategy for finding limits.
- Evaluate a limit using dividing out and rationalizing techniques.
- Evaluate a limit using the Squeeze Theorem.

THEOREM 1.1 SOME BASIC LIMITS

Let b and c be real numbers and let n be a positive integer.

1. $\lim\limits_{x \to c} b = b$ **2.** $\lim\limits_{x \to c} x = c$ **3.** $\lim\limits_{x \to c} x^n = c^n$

HEOREM 1.2 PROPERTIES OF LIMITS

et b and c be real numbers, let n be a positive integer, and let f and g be nctions with the following limits.

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = K$$

Scalar multiple: $\lim_{x \to c} [b f(x)] = bL$

Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$

Product: $\lim_{x \to c} [f(x)g(x)] = LK$

Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$, provided $K \neq 0$

Power: $\lim_{x \to c} [f(x)]^n = L^n$

Strategy for finding limits

1. Direct Substitution
2. Algebraic techniques

(factoring or rationalizing or simplifying)

3. Special Cases

THEOREM 1.9 TWO SPECIAL TRIGONOMETRIC LIMITS
1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$ **2.** $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$

Ex 1

$$\lim_{x\to 2} \frac{\sqrt{x+2}}{x-4}$$

$$\frac{\sqrt{4}}{-2}$$

$$-1$$

Ex 2

$$\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$$

$$\sec \frac{7\pi}{6}$$

$$-\frac{2\sqrt{3}}{3} \text{ or}$$

$$-\frac{2}{\sqrt{3}}$$

Ex 3

$$\lim_{x \to 5\pi/3} \cos x$$

$$\frac{1}{2}$$

Ex 4

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(2x-3)}{\cancel{x+1}}$$

$$-5$$

Ex 5

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{x+1}}$$

$$(-1)^2 - (-1) + 1$$

$$3$$

Ex 6

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{\cancel{x+1-4}}{\cancel{(x-3)}(\sqrt{x+1}+2)}$$

$$\lim_{x\to 3} \frac{1}{\sqrt{x+1}+2}$$

$$\frac{1}{4}$$

Ex 7

$$\lim_{x\to 0} \frac{[1/(x+4)] - (1/4)}{x}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$$

$$\lim_{x\to 0} \frac{\frac{4-(x+4)}{4(x+4)}}{x}$$

$$\lim_{x\to 0} \frac{\frac{-x}{4(x+4)}}{\frac{x}{1}}$$

$$\lim_{x\to 0} \frac{-x}{4(x+4)} \cdot \frac{1}{x}$$

$$\lim_{x\to 0} \frac{-1}{4(x+4)}$$

$$\frac{-1}{16}$$

Ex 7

$$\lim_{x\to 0} \frac{[1/(x+4)] - (1/4)}{x}$$

LCD: $(x+4)4$

$$\lim_{x\to 0} \frac{4(x+4)\left(\frac{1}{x+4}\right) - \left(\frac{1}{4}\right)4(x+4)}{(x)(4(x+4))}$$

$$\lim_{x\to 0} \frac{4-(x+4)}{4x(x+4)}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{4\cancel{x}(x+4)} = -\frac{1}{16}$$

Ex 8

$$\lim_{x\to 0} \frac{3(1-\cos x)}{x}$$

$3 \cdot 0$

0

$$3\left[\lim_{x\to 0} \frac{1-\cos x}{x}\right]$$

Ex 9

$$\lim_{t\to 0} \frac{\sin(3t)}{(2t)}$$

$$\frac{1}{2} \lim_{t\to 0} \frac{\sin(3t)}{t} \cdot \frac{3}{3}$$

$$\frac{3}{2}\left[\lim_{t\to 0} \frac{\sin 3t}{3t}\right]$$

$$\frac{3}{2} \cdot 1 = \frac{3}{2}$$

$$\lim_{x \to 0} \frac{\sin 4x}{11x}$$

$$\frac{1}{11} \lim_{x \to 0} \frac{\sin 4x}{x} \cdot \frac{4}{4}$$

$$\frac{4}{11} \left[\lim_{x \to 0} \frac{\sin 4x}{4x} \right]$$

$$\frac{4}{11}$$

Ex 10: Given f(x) = 5x - 2,

$f(x+\Delta x) = 5(x+\Delta x) - 2$

find $\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$.

$$\lim_{\Delta x \to 0} \frac{5(x+\Delta x) - 2 - (5x-2)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{5\cancel{\Delta x}}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} 5 = 5$$

Ex. 11

$$\lim_{x \to c} f(x) = \frac{3}{2}$$

$$\lim_{x \to c} g(x) = \frac{1}{2}$$

(a) $\lim_{x \to c} [4f(x)] = 4\left(\frac{3}{2}\right) = 6$

(b) $\lim_{x \to c} [f(x) + g(x)] = \frac{3}{2} + \frac{1}{2} = 2$

(c) $\lim_{x \to c} [f(x)g(x)] = \frac{3}{4}$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)} = 3$