1.2 Finding Limits Graphically and Numerically

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

1. Numerical approach	Construct a table of values.
2. Graphical approach	Draw a graph by hand or using technology.
3. Analytic approach	Use algebra or calculus.

Ex 1

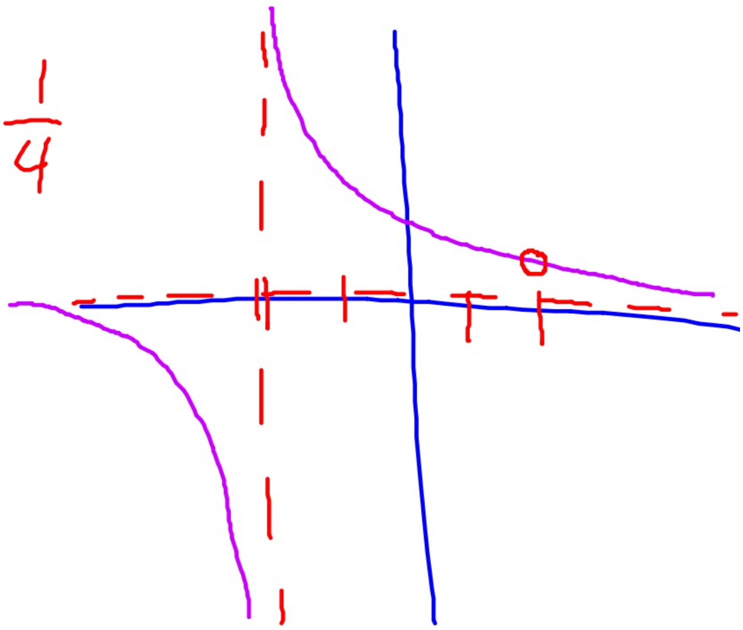
$$\lim_{x \to 2} \frac{x-2}{x^2-4} = .25$$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	.256	.251	.250	.2499	.2493	.2439

$\frac{0}{0}$: indeterminate form

$$\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

graphical approach



$$f(x) = \frac{x-2}{x^2-4}$$

$$= \frac{x-\cancel{2}}{(\cancel{x-2})(x+2)}$$

$$= \frac{1}{x+2}$$

hole $\left(2, \frac{1}{4}\right)$

Ex 2

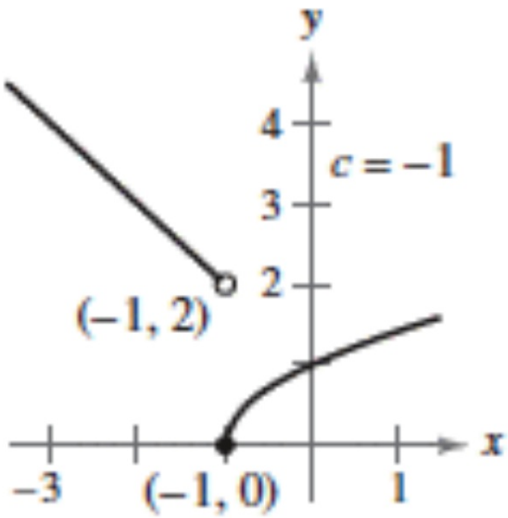
$$\lim_{x \to 4} \frac{[x/(x+1)] - (4/5)}{x - 4} = .04$$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$	.0408	.04008	.04001	.03999	.03992	.03922

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of c than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as x approaches c.
3. $f(x)$ oscillates between two fixed values as x approaches c.

DNE
Type #1

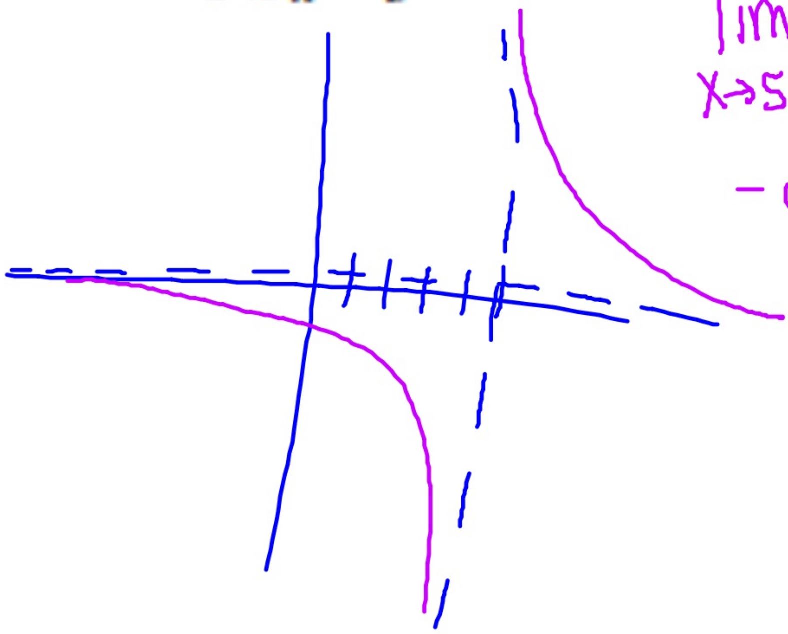


$$\lim_{x \to -1^-} g(x) \neq \lim_{x \to -1^+} g(x)$$

Justification for DNE

DNE Type #2

2. $\lim_{x \to 5} \frac{2}{x-5}$



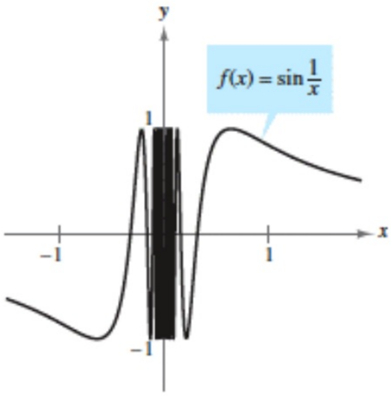
$\lim_{x \to 5^-} \frac{2}{x-5} \neq \lim_{x \to 5^+} \frac{2}{x-5}$

$-\infty \qquad \infty$

DNE Type #3

3.

$f(x) = \sin\left(\frac{1}{x}\right)$



$\lim_{x \to 0} f(x)$ does not exist.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	.544	.506	−.827	.827	−.506	−.544

DNE

(a) $f(-2)$ undefined

(b) $\lim\limits_{x \to -2} f(x)$ dne

(c) $f(0) = 4$

(d) $\lim\limits_{x \to 0} f(x)$ dne

(e) $f(2)$ undefined

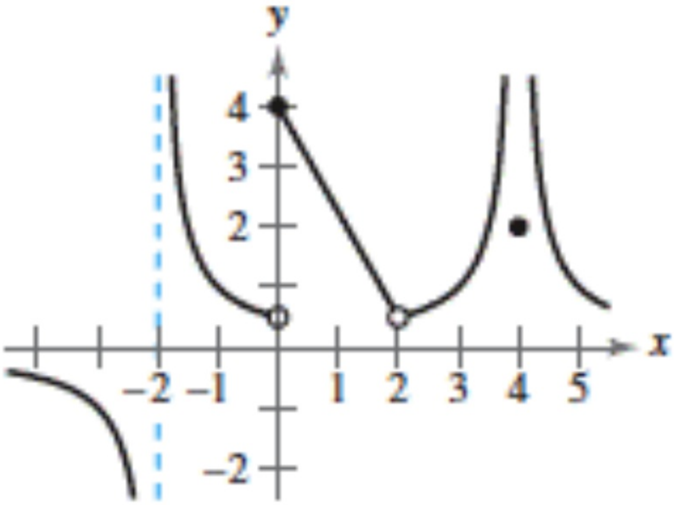
(f) $\lim\limits_{x \to 2} f(x) = \frac{1}{2}$

(g) $f(4) = 2$

(h) $\lim\limits_{x \to 4} f(x)$ $\boxed{\infty}$ or dne

more specific

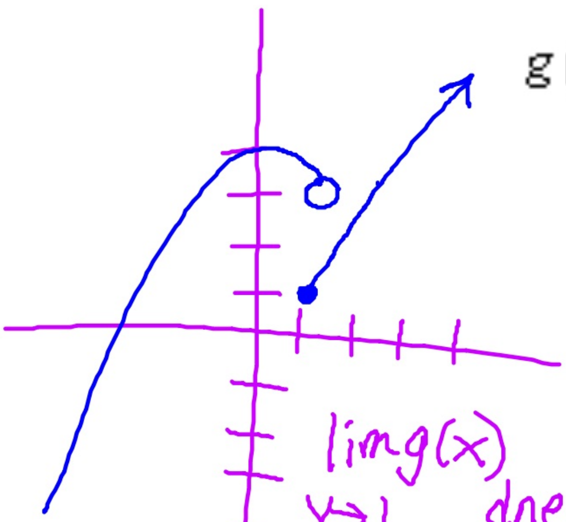
∞ is a special case of dne



Sketching Piecewise functions

Sketch the graph of the following piecewise function.

$$g(x) = \begin{cases} -x^2+4 & \text{if } x<1 \\ 2x-1 & \text{if } x\geq 1 \end{cases}$$



$\lim_{x\to 1} g(x)$ dne

$\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$

$\lim_{x\to 1^-} g(x) = 3$

$\lim_{x\to 1^+} g(x) = 1$

Sketch the graph of the following piecewise function.

$$h(x)=\begin{cases} x+3 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x<1 \\ -x+2 & \text{if } x\ge 1 \end{cases}$$

In Exercises 31 and 32, sketch a graph of a function f that satisfies the given values. (There are many correct answers.)

$$f(-2) = 0$$

$$f(2) = 0$$

$$\lim_{x \to -2} f(x) = 0$$

$$\lim_{x \to 2} f(x) \text{ does not exist.}$$

(a) If $f(2) = 4$, can you conclude anything about the limit of $f(x)$ as x approaches 2? Explain your reasoning.

(b) If the limit of $f(x)$ as x approaches 2 is 4, can you conclude anything about $f(2)$? Explain your reasoning.