7.2: Volume by cross section

You will be given:

- base (determined by region enclosed by functions)
- geometric shape
- perpendicular to x-axis or y-axis

$\perp$ x-axis

$$\int_a^b \text{Area}\,dx$$

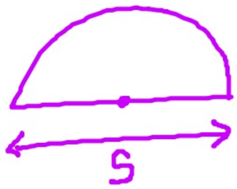
$\perp$ y-axis

$$\int_a^b \text{Area}\,dy$$

Geometric Shapes

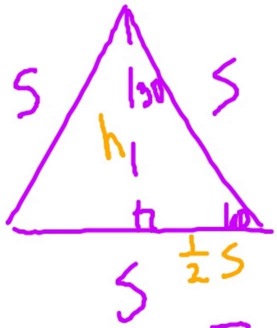
Square s^2

Rectangle lw



$\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi\left(\frac{s}{2}\right)^2$



$h = \frac{\sqrt{3}}{2}s$

$A = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right)$

$A = \frac{\sqrt{3}}{4}s^2$

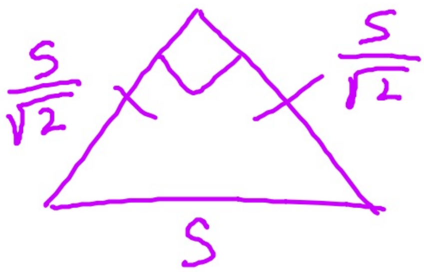
Equilateral triangle $\frac{\sqrt{3}}{4}s^2$

Semi-circles $\frac{\pi}{8}s^2$

Isosceles Right Triangles (hypotenuse on base) $\frac{1}{4}s^2$

Isosceles Right Triangles (leg on base) $\frac{1}{2}s^2$

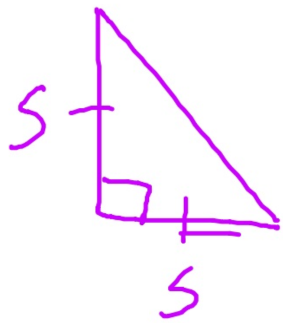
hyp. on base



$$\frac{1}{2} \cdot \frac{s}{\sqrt{2}} \cdot \frac{s}{\sqrt{2}}$$

$$\frac{1}{4} s^2$$

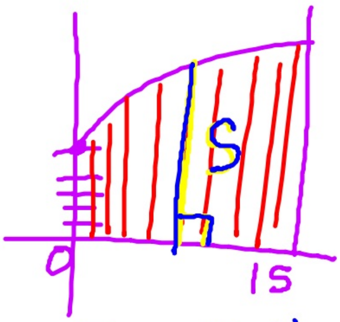
leg on base



$$\frac{1}{2} s^2$$

① $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

$\perp$ x-axis (squares)



$S = \text{Top} - \text{Bottom}$

$S = (2\sqrt{x} + 5 - 0)$

$$\int_0^{15} \text{Area}\, dx = \int_0^{15} S^2\, dx$$

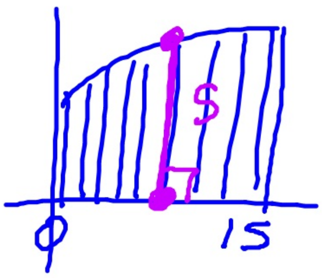
$$\int_0^{15} (2\sqrt{x} + 5)^2\, dx = 1599.596$$

or

1599.597

(2) $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

$\perp$ x-axis semicircle

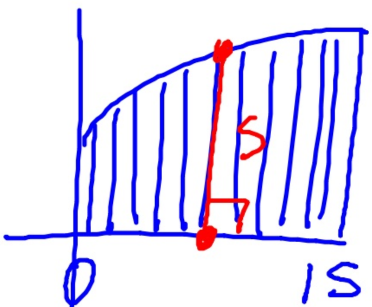


$S = 2\sqrt{x} + 5$

$$\frac{\pi}{8}\int_0^{15} \left(2\sqrt{x} + 5\right)^2 dx = 628.160$$

③ $f(x) = 2\sqrt{x} + 5 \quad [0, 15]$

$\perp$ x-axis rectangle of height 4



$s = 2\sqrt{x} + 5$

$lw = (2\sqrt{x} + 5)4$

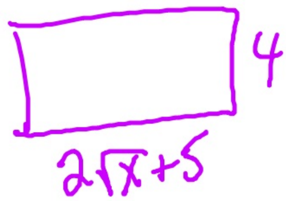
$s \cdot 4$

$$\int_0^{15} \text{Area}\,dx = \int_0^{15} (2\sqrt{x}+5)4\,dx$$

609.838

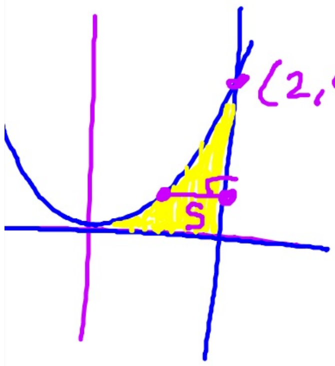
or

609.839



④ Base enclosed by $y = x^2$, $y = 0$, and $x = 2$

$\perp$ y-axis semicircles



$$\frac{\pi}{8}\int_0^4 (2-\sqrt{y})^2\,dy$$

1.047

$\sqrt{y} = \sqrt{x^2}$

$\sqrt{y} = x$

$S = \text{Right} - \text{left}$

$S = 2 - \sqrt{y}$

Let R be the region in the first and second quadrant bounded above by the graph of $y=\frac{20}{1+x^2}$ and below by the horizontal line y = 2.

2007: 2.74

The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume.

$$\frac{\pi}{8}\int_{-3}^{3}\left(\frac{20}{1+x^2}-2\right)^2 dx = 174.268$$