5.4 Exponential Functions: Differentiation and Integration

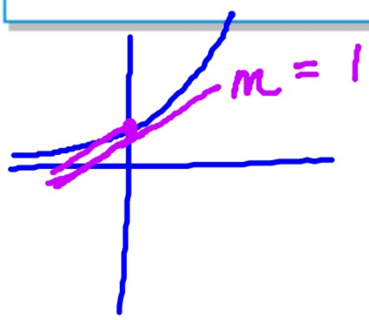
DEFINITION OF THE NATURAL EXPONENTIAL FUNCTION

The inverse function of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and is denoted by

$$f^{-1}(x) = e^x.$$

That is,

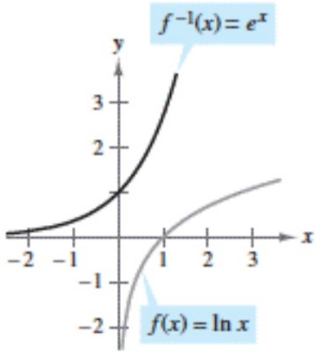
$$y = e^x \qquad \text{if and only if} \qquad x = \ln y.$$



$y = e^x$

$y' = e^x$

$y'(0) = 1$



The inverse function of the natural logarithmic function is the natural exponential function.

$$\ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

THEOREM 5.11 DERIVATIVES OF THE NATURAL EXPONENTIAL FUNCTION

Let u be a differentiable function of x.

1. $\frac{d}{dx}[e^x] = e^x$
2. $\frac{d}{dx}[e^u] = e^u \frac{du}{dx}$

$y = e^u$

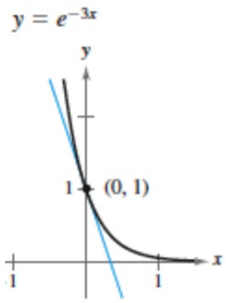
$y' = e^u \cdot u'$

$y = e^{7x^2 - 5x}$

$y' = e^{7x^2-5x}(14x-5)$

$(\ln x)^2 \neq \ln x^2$

#1: Find the equation of the tangent line at the given point



$y = e^{-3x}$

$y' = -3e^{-3x}$

$y'(0) = -3$

$y - 1 = -3(x - 0)$

#2: Find y'

$y = x^2 e^{-x}$

$y' = x^2 \cdot -e^{-x} + e^{-x} \cdot 2x$

$y' = -xe^{-x}(x - 2)$

#3: Find y'.

$$y = \frac{e^{2x}}{e^{2x}+1}$$

$$y' = \frac{(e^{2x}+1)2e^{2x} - e^{2x}(2e^{2x})}{(e^{2x}+1)^2}$$

$$y' = \frac{2e^{2x}(\cancel{e^{2x}}+1-\cancel{e^{2x}})}{(e^{2x}+1)^2}$$

$$y' = \frac{2e^{2x}}{(e^{2x}+1)^2}$$

#4: Find dy/dx

$$\frac{d}{dx}\left(e^{xy} + x^2 - y^2 = 10\right)$$

$$e^{xy}\left(x\frac{dy}{dx} + y \cdot 1\right) + 2x - 2y\frac{dy}{dx} = 0$$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} + 2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x - ye^{xy}}{xe^{xy} - 2y}$$

Integration for functions of e

$$\int e^x\,dx = e^x + C$$

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$$\int e^{2x}\,dx = \frac{1}{2}\int e^u\,du$$

$u = 2x$
$du = 2\,dx$

$$= \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{2x} + C$$

#6 $\frac{1}{2}\int \frac{2 \cdot e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

$u = 1 + e^{2x}$

$du = 2e^{2x}\,dx \qquad = \frac{1}{2}\ln|1+e^{2x}| + C$

$\ln ab = \ln a + \ln b$

#7 $\int_0^1 \frac{e^x}{5-e^x}\,dx = \int_0^1 \frac{e^x}{-(e^x-5)}\,dx$

$-\int_0^1 \frac{e^x}{e^x-5}\,dx = -\int \frac{1}{u}\,du = -\ln|e^x-5|\Big]_0^1$

$u = e^x - 5$

$du = e^x\,dx$

$-\left[\ln|e-5| - \ln 4\right]$

$-\ln\frac{|e-5|}{4}$

$\ln\frac{4}{|e-5|} = \ln\frac{4}{5-e}$

$\int \frac{e^x+7}{e^x}\,dx$

$\int 1 + 7e^{-x}\,dx$