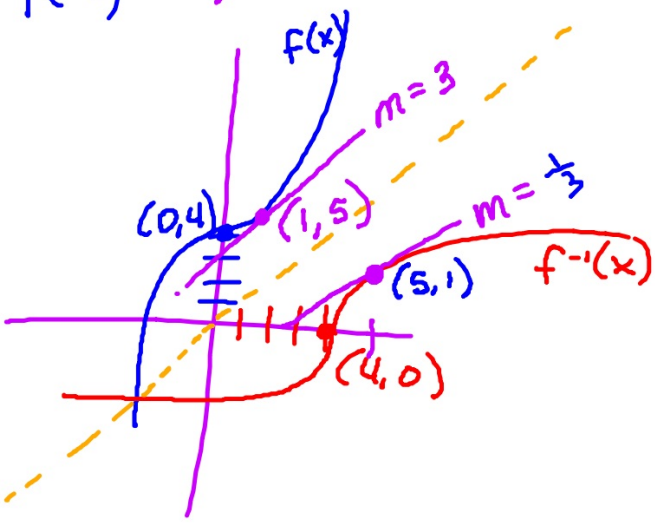
# 5.3 Inverse Functions

$f(x) = x^3 + 4$

$f^{-1}(x) = \sqrt[3]{x-4}$




$f'(x) = 3x^2$

$f'(1) = 3$

$(f^{-1})'(x) = \frac{1}{3}(x-4)^{-2/3} \cdot 1$

$(f^{-1})'(5) = \frac{1}{3}(5-4)^{-2/3}$

$= \frac{1}{3}$

① $f(x) = 2x^3 + 3x$ $\quad (f^{-1})'(5) =$ _____

Verify the function is 1:1 (monotonic)
Is f(x) always increasing or always decreasing?

$f'(x) = 6x^2 + 3$

$0 = 6x^2 + 3$

$\xleftrightarrow{+} f'$

f(x) is always increasing because $f' > 0$
f(x) is monotonic

$f : (1, 5)$

$f^{-1} : (5, 1)$

$5 = 2x^3 + 3x$

$1 = x$

$f'(x) = 6x^2 + 3$

$f'(1) = 9$

$(f^{-1})'(5) = \frac{1}{9}$

$(f^{-1})'(5) = \frac{1}{f'(1)}$

reciprocal

Suppose that **f** is a function which is differentiable on the open interval **I**. If either **f** '(**x**) > **0** or **f** '(**x**) < **0** for all **x** in **I** then f which is defined and is differentiable on **f**(**I**). That is, for each **a** in **I**, $\mathbf{f}^{-1}$ is differentiable at **b** = **f**(**a**) and

$$(f^{-1})'(b) = \frac{1}{f'(a)}$$

(2) $f(x) = x^3 - \frac{4}{x}$ D: $(0, \infty)$

If f(x) and g(x) are inverses, find g'(6)

$f: (2, 6)$

$g: (6, 2)$

$6 = x^3 - \frac{4}{x}$

$2 = x$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$f'(2) = 13$

$g'(6) = \frac{1}{13}$

21. 14% correct

Let $f$ be a differentiable function such that $f(3) = 15$, $f'(3) = -8$, and $f'(6) = -2$, $f(6) = 3$

The function $g$ is differentiable and $g(x) = f^{-1}(x)$ for all $x$. What is the value of $g'(3)$?

A) $-\frac{1}{2}$ B) $-\frac{1}{8}$ C) $\frac{1}{6}$ D) $\frac{1}{3}$ E) The value of $g'(3)$ cannot be determined from the information given.

$f'(6) = -2$

reciprocal

$g: (3, \quad)$

$f: (6, 3)$

AP Question. Mean Score 0.95

The functions $f$ and $g$ are differentiable for all real numbers, and $g$ is strictly increasing. The table gives values of the functions and their first derivatives at selected values of $x$. The function $h$ is given by $h(x) = f(g(x)) - 6$.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 6 | 4 | 2 | 5 |
| 2 | 9 | 2 | 3 | 1 |
| 3 | 10 | -4 | 4 | 2 |
| 4 | -1 | 3 | 6 | 7 |

c.) $w'(x) = f(g(x)) \cdot g'(x)$

$w'(3) = f(g(3)) \cdot g'(3)$

$(-1)(2)$

$w'(3) = -2$

(a) Explain why there must be a value $r$ for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value $c$ for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let $w$ be the function given by $w(x) = \int_1^{g(x)} f(t)dt$. Find the value $w'(3)$.

(d) If $g^{-1}$ is the inverse function of $g$, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

$(2, 1)$

$g'(1) = 5 \rightarrow$ reciprocal

$y - 1 = \frac{1}{5}(x - 2)$

$g^{-1} : (2, 1)$

$g : (1, 2)$

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b) $\frac{h(3)-h(1)}{3-1} = \frac{-7-3}{2}$

$= -5$

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a) By MVT, there must exist a value c such that h'(c) = -5 because