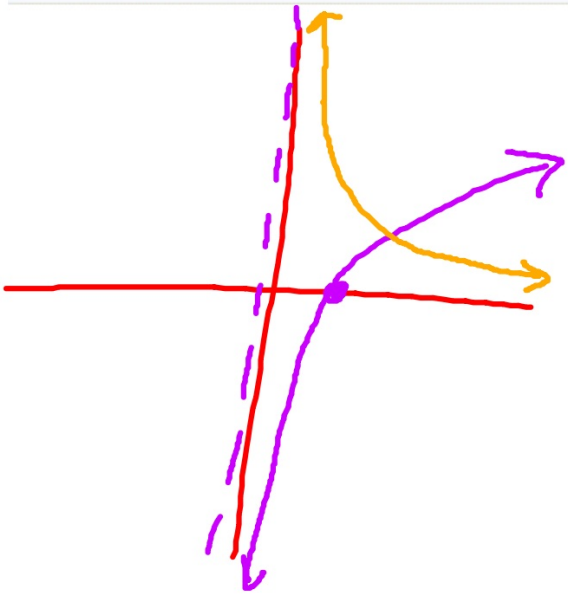
# 5.1 The Natural Logarithmic Function: Differentiation

- Develop and use properties of the natural logarithmic function.
- Understand the definition of the number $e$.
- Find derivatives of functions involving the natural logarithmic function.



$$y = \ln x, \quad x > 0$$

$$y' = \frac{1}{x}$$

Derivative of y = lnx

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0$$

Derivative of y = ln u

$$\frac{d}{dx}[\ln u] = \frac{1}{u} \cdot u', \quad u > 0$$

$\frac{u'}{u}$

$y = \ln(2x+1)$

$y' = \frac{1}{2x+1} \cdot 2 = \frac{2}{2x+1}$

$$\ln 1 = 0$$

$$\ln e = 1$$

$$e^{\ln x} = x$$

Find the derivative.

#1 $h(t) = \dfrac{\ln t}{t}$ $t>0$

$h'(t) = \dfrac{t\cdot\frac{1}{t} - \ln t \cdot 1}{t^2}$

$h'(t) = \dfrac{1-\ln t}{t^2}$

Rel. extrema for $h(t)$?

$0 = 1 - \ln t$ (+ | 0 | + | e | −)

$e^{\ln t} = e^1$ $t = e$ rel. max $\left(e, \frac{1}{e}\right)$

#2 $y = \ln \sqrt[3]{\dfrac{x-1}{x+1}}$

$y = \frac{1}{3}\ln(x-1) - \frac{1}{3}\ln(x+1)$

$y' = \frac{1}{3}\cdot\frac{1}{x-1} - \frac{1}{3}\cdot\frac{1}{x+1}$

$y' = \dfrac{1}{3(x-1)} - \dfrac{1}{3(x+1)}$

**THEOREM 5.4 DERIVATIVE INVOLVING ABSOLUTE VALUE**

If $u$ is a differentiable function of $x$ such that $u \neq 0$, then

$$\frac{d}{dx}[\ln|u|] = \frac{u'}{u}$$

#3 $f(x) = \ln|\cos x|.$

$$f'(x) = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

#4: Find the relative extrema. Justify. $x > 0$

$$y = x - \ln x$$

$$y' = 1 - \frac{1}{x} = \frac{x-1}{x}$$

$$x = 1$$


rel. min $(1, 1)$ because $y'$ changes from $-/+$

#5: Find dy/dx using implicit differentiation.

$$4xy + \ln x^2y = 7$$

$\frac{d}{dx}\left(4xy + 2\ln x + \ln y = 7\right)$

$4\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + \frac{2}{x}\frac{dx}{dx} + \frac{1}{y}\frac{dy}{dx} = 0$

$4x\frac{dy}{dx} + 4y + \frac{2}{x} + \frac{1}{y}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{\left(\frac{-2}{x} - 4y\right)xy}{\left(4x + \frac{1}{y}\right)xy} = \frac{-2y - 4xy^2}{4x^2y + x}$

#6: Find the equation of the tangent line at the given point.

$$f(x) = 3x^2 - \ln x, \quad (1, 3)$$

$f'(x) = 6x - \frac{1}{x}$

$f'(1) = 5$

$y - 3 = 5(x - 1)$

7) $f(x) = \ln\left(x\sqrt{x^2+1}\right)$

$f(x) = \ln x + \ln\sqrt{x^2+1}$

$f(x) = \ln x + \frac{1}{2}\ln(x^2+1)$

$f'(x) = \frac{1}{x} + \frac{1}{2}\cdot\frac{2x}{x^2+1}$ SKIP 101

$f'(x) = \frac{1}{x} + \frac{x}{x^2+1}$