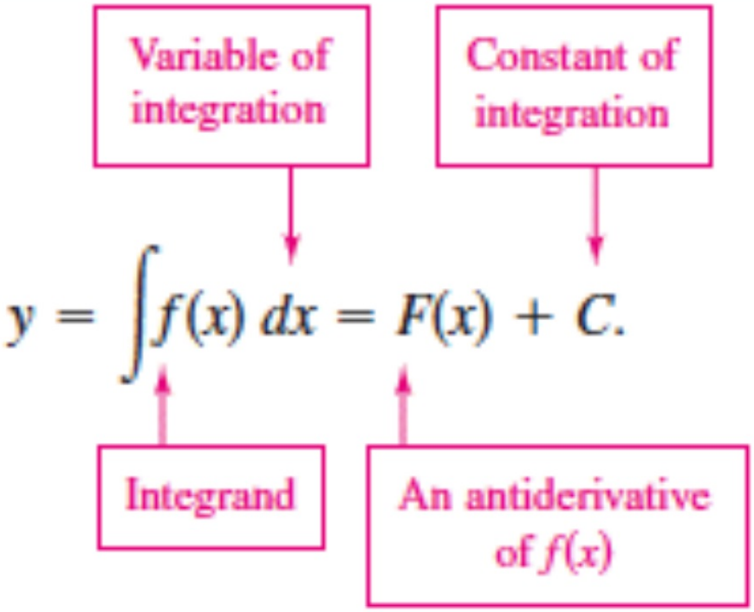
4.1 Antiderivatives and Indefinite Integration

- Write the general solution of a differential equation.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Find a particular solution of a differential equation.



If you know f'(x), how can you find f(x)?

$$y = x^2 + 3$$

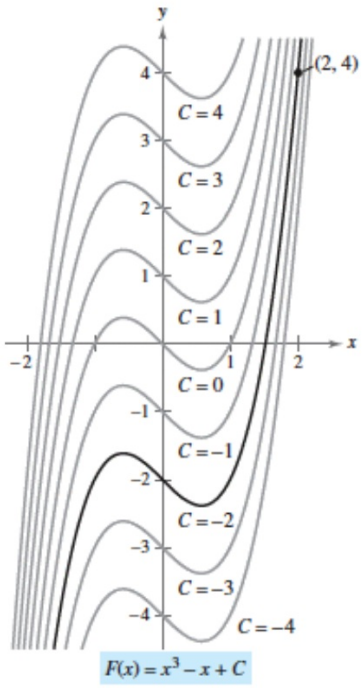
$$y' = 2x$$

$$\int 2x\,dx = x^2 + C$$

f'(x) = $3x^2-1$
Integrate to
find f(x)

$$\int (3x^2 - 1)\,dx$$

$$x^3 - x + C$$



$f(x) = x^n$

$f'(x) = n x^{n-1}$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$n \neq -1$

$y = 10x^2$

$y' = 20x$

$\int 20x\,dx$

$\frac{20x^2}{2}$

#1 $\int (8x^3 - 9x^2 + 4)\,dx$

$= \frac{8x^4}{4} - \frac{9x^3}{3} + 4x + C$

$= 2x^4 - 3x^3 + 4x + C$

#2 $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right) dx$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

#3 $\int \frac{x^2 + 2x - 3}{x^4}\,dx = \int \left(x^{-2} + 2x^{-3} - 3x^{-4}\right)dx$

$$\frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C$$

$$-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$$

#4 $\int (2t^2 - 1)^2\,dt = \int \left(4t^4 - 4t^2 + 1\right)dt$

$$= \frac{4t^5}{5} - \frac{4t^3}{3} + t + C$$

Integration Rules

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$ Power Rule

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$\int(\sec^2 x - \sin x)\,dx$

$\tan x + \cos x + C$

#5 $\int \sec y(\tan y - \sec y)\,dy = \int (\sec y \tan y - \sec^2 y)\,dy$

$= \sec y - \tan y + C$

#6 $\int \frac{\sin x}{1 - \sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \frac{\sin x \cdot 1}{\cos x \cdot \cos x}\,dx$

$= \int \tan x \sec x\,dx = \sec x + C$

$$\int (x+1)\,dx = \int x\,dx + \int 1\,dx$$

$$= \frac{x^2}{2} + C_1 + x + C_2$$

$$\frac{x^2}{2} + x + C$$