3.5 Limits at Infinity

- Determine (finite) limits at infinity.
- Determine the horizontal asymptotes, if any, of the graph of a function.
- Determine infinite limits at infinity

Limits at Infinity

This section discusses the "end behavior" of a function on an *infinite* interval.

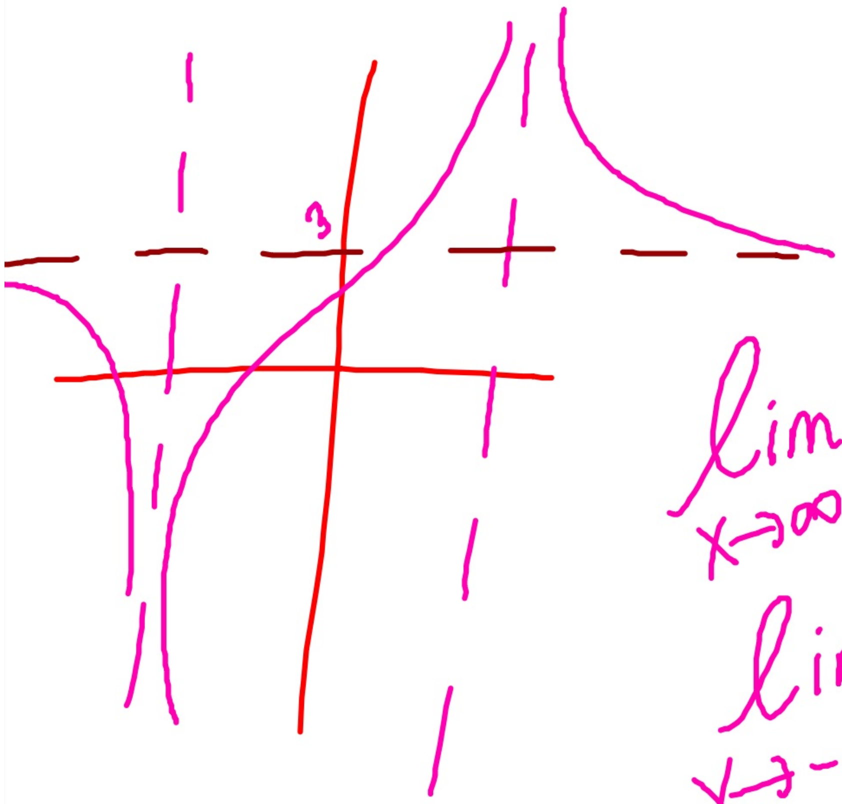
Horizontal Asymptotes

In Figure 3.34, the graph of f approaches the line $y = L$ as x increases without bound. The line $y = L$ is called a **horizontal asymptote** of the graph of f.

DEFINITION OF A HORIZONTAL ASYMPTOTE

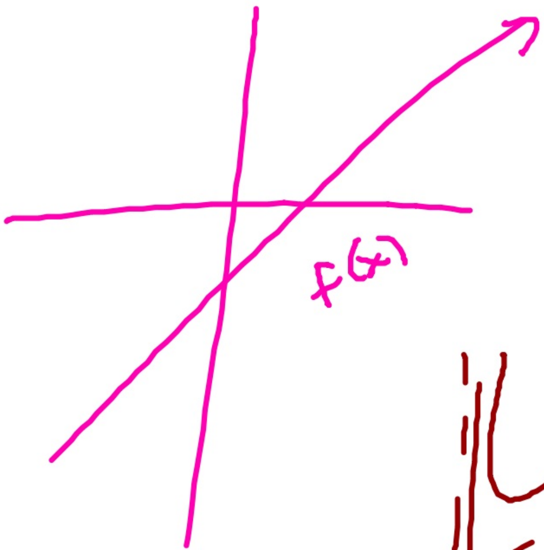
The line $y = L$ is a **horizontal asymptote** of the graph of f if

$$\lim_{x\to-\infty} f(x) = L \quad \text{or} \quad \lim_{x\to\infty} f(x) = L.$$

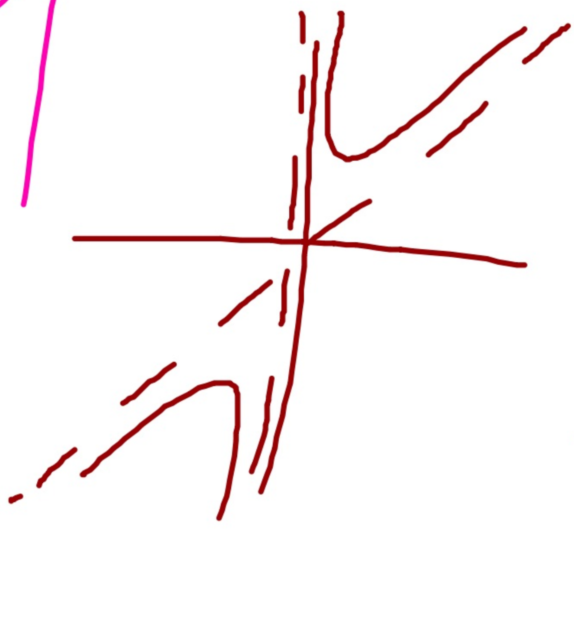


$$\lim_{x \to \infty} f(x) = 3$$

$$\lim_{x \to -\infty} f(x) = 3$$



$$\lim_{x \to \infty} f(x) = \infty$$

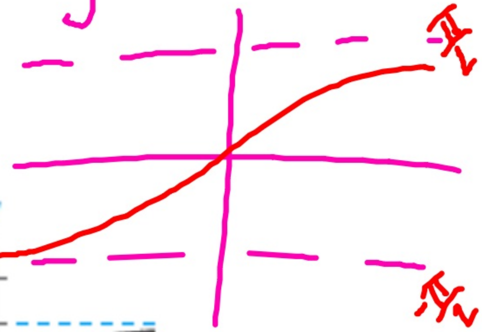


$$\lim_{x \to \infty} g(x) = \infty$$

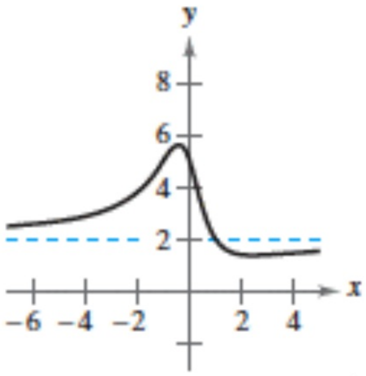
$$\lim_{x \to -\infty} g(x) = -\infty$$

Find the limit as x approaches infinity and negative infinity

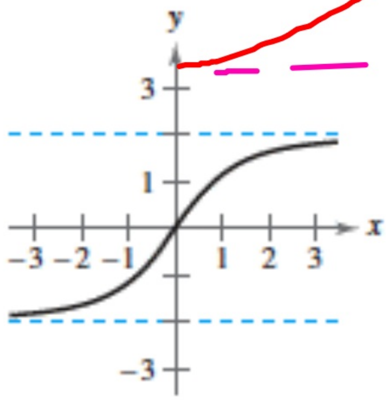
y=arctanx



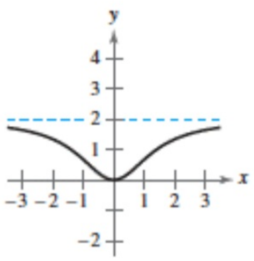
Ex 1



Ex 2



Ex 3



Ex 4

(a) $\lim_{x\to\infty} \frac{3-2x}{3x^3-1} = 0$ BoBo

(b) $\lim_{x\to\infty} \frac{3-2x}{3x-1} = -\frac{2}{3}$ eatsdc

(c) $\lim_{x\to\infty} \frac{3-2x^2}{3x-1} = -\infty$ Botn

no HA.

none

GUIDELINES FOR FINDING LIMITS AT $\pm\infty$ OF RATIONAL FUNCTIONS

1. If the degree of the numerator is *less than* the degree of the denominator, then the limit of the rational function is 0.
2. If the degree of the numerator is *equal to* the degree of the denominator, then the limit of the rational function is the ratio of the leading coefficients.
3. If the degree of the numerator is *greater than* the degree of the denominator, then the limit of the rational function does not exist.

Find the limit.

Ex 5

$$\lim_{x\to-\infty}\left(\frac{5}{x}-\frac{x}{3}\right) = \lim_{x\to-\infty} \frac{15-x^2}{3x} = +\infty$$

Ex 6

$$\lim_{x\to-\infty}\left(\frac{1}{2}x-\frac{4}{x^2}\right) = \lim_{x\to-\infty} \frac{x^3-8}{2x^2} \quad \frac{-\text{Bigger}}{+\text{Big}}$$

$$\lim_{x\to-\infty} \frac{x}{2} - \lim_{x\to-\infty} \frac{4}{x^2}$$

$$\frac{4}{x^2} = \frac{4}{10000000} \quad -\infty$$

$$-\text{Big} - 0$$

$$\frac{1}{\text{Big}} = 0$$

$$\frac{1}{\text{small}} = \infty$$

Ex 7

$\lim_{x\to\infty} \sin x$ dne

Ex 8

$\lim_{x\to\infty} \frac{\sin x}{x}$ $\frac{\pm 1}{\text{Big}} = 0$

Ex 10

Find the two horizontal asymptotes for:

$$f(x) = \frac{20x}{\sqrt{9x^2 - 1}}$$

$$\lim_{x \to \infty} \frac{20x}{\sqrt{9x^2-1}} \qquad \lim_{x \to -\infty} \frac{20x}{\sqrt{9x^2-1}}$$

$$\frac{20}{3} \qquad \frac{-20}{3}$$

$$\lim_{x \to \infty} \frac{5x}{\sqrt{x^2+1}} \qquad \lim_{x \to -\infty} \frac{5x}{\sqrt{x^2+1}}$$

$$5 \qquad\qquad -5$$

$$\lim_{x \to -\infty} \frac{-3x}{\sqrt{9x^2+5}} = 1$$